

Meios Computacionais no Ensino (M.C.E.)

Prof. Jaime Carvalho e Silva

Universidade de Coimbra

2º Trabalho

Andreas Najock

Exercise:

Find 20 solutions for geometrical problems on the site
<http://www.polarprof.org/geometriagon/> .

Answers:

My username on the website is Murkel and all solutions are described on the website too. There can also be seen the images of the constructions. Below is a list of the solved problems with number, description and information's.

1. Number of Problem: 1

Difficulty: 2.1

Problem: Given three points A, B, C draw a circle centered on C with radius AB

Short solution: Draw the perpendicular bisector of AC. And reflect the intersection of a circle around A through B with the line through AC to get a distance from C with the length of AB.

Construction: Draw a line through A and C and a circle around A through B and mark the intersection below T.

Construct the perpendicular bisector of A and C by connecting the intersections of the circles around A through C and around C through A.

Now reflect the point T at the perpendicular bisector of A and C by drawing a circle around the intersection of this perpendicular bisector and AB (marked as Z) through T.

The other intersection with the line through A and C mark as U. draw a circle around C through U.

2. Number of Problem: 10

Difficulty: 1.2

Problem: Given two points A and B, construct the (red) circle through A and B with the given radius OP

Short solution: Use two circles around to find the center of the circle.

Construction: Draw two circle with radius r around A and B.

Mark the intersection of the two circles with M.

Draw a circle around M through A.

3. Number of Problem: 11

Difficulty: 1.4

Problem: Given a point A and a line, construct the (red) circle through A with given radius OP and tangent to the line

Short solution: Use a circle with radius r around A and create the parallel to the line with distance r, the intersection is the center of the circle.

Construction: Draw a circle around A with radius r.

Draw a circle around a random point P on the line with radius r.

Draw the orthogonal to the line through P.

Mark the intersection with the circle around P as T.

Draw a parallel to the line through T.

The intersection of the parallel with the circle around A is the center of the wanted circle (M).

Draw a circle around M through A.

4. Number of Problem: 12

Difficulty: 1.5

Problem: Given two intersecting lines, construct the (red) circle with given radius OP, which is tangent to both lines

Short solution: Draw two parallels with distance r to the lines. The intersection is the center of the circle.

Construction: Draw a circle with radius r through the intersection of the lines P .

Draw the orthogonal lines to the lines through P .

Draw the parallels to the lines through the intersections of the circle to the orthogonal lines of the respective lines. (Z_1 and Z_2)

Mark the intersection of the two parallels as M . This is the center of the circle so draw the circle with radius r around M .

5. Number of Problem: 13

Difficulty: 1.5

Problem: Given a point A and a circle, construct the (red) circle through A with given radius OP , and tangent to the given circle

Short solution: Use a point on the tangent circle to create the radius $r+r(\text{tangent})$. And with a circle of that radius around U and a circle around A with radius r find the center of the circle.

Construction: Draw a circle around A with radius r .

Select a point P on the tangent circle and draw a line through P and U .

Draw a circle around P with radius r and mark the intersection with the line t with Z .

Draw a circle around U with radius=length of UZ .

Intersection of this circle and the circle around A is the center M of the CIRCLE.

Draw a circle around M with radius r

6. Number of Problem: 14

Difficulty: 1.6

Problem: Construct the (red) circle with given radius r which is tangent to the two given circles

Short solution: Use point on tangent circle to create radius $r+r(\text{tangent})$.

Repeat two times to get the center.

Construction: Repeat the method of the tangent circle in 13 for the two tangent circles to get the distance to the center of the circle that is asked for (M).

Draw the circle around M with radius r.

7. Number of Problem: 15

Difficulty: 1.7

Problem: Construct the (red) circle with given radius r which is tangent to both the given line and the given circle

Short solution: Use parallel to line with distance r and find a circle with radius $r+r(\text{tangent})$ around U to find the center of the circle.

Construction: Pick a point K on the line and draw a circle with radius r and the orthogonal to the line through K.

Draw a parallel to the line through the intersection of circle and orthogonal (O).

Pick a point P on tangent circle and draw a line through U and P.

Draw a circle with radius r around P and mark the intersection of circle and line as Z.

Draw a circle around U through Z and mark the intersection with the parallel with M.

M is the center of the circle, so draw a circle around M with radius r.

8. Number of Problem: 19

Difficulty: 1.5

Problem: Given a line and points A and B, find points C and D on the line such that the angles CAD and CBD are right angles.

Short solution: Find a proper circle to use theorem of thales. Therefore its necessary to find the point on the given line, which has the same distance to A and B.

Construction: Draw a circle around A through B.

Draw a circle around B through A and draw a line through the intersections of the two circles (I1 and I2).

Mark the intersection of this line with the given line as P.

Draw a circle around P through A.

The intersections of this circle with the given line are the points C and D

9. Number of Problem: 25

Difficulty: 1.9

Problem: Find on line **a** the (red) point equidistant from a given point A on the line and another given line **b**

Short solution: Use angle bisectors to find two parallels that are orthogonal to **b**.

Construction: Draw the orthogonal to **b** through A.

Construct the angle bisector of this line and **a** by drawing a circle around A, marking the midpoint (Z) of the two intersections with the orthogonal and **a** and draw a line through Z and A.

Mark the intersection of this angle bisector and **b** as J.

Draw the orthogonal to **b** through J.

The intersection of this line through **a**, is the point P that is looked for.

10. Number of Problem: 29

Difficulty: 1.4

Problem: Given **A**, **c**, & **b-c**, angle A, and the lengths of AB, and AC-BC (d), find vertex C of triangle ABC.

Short solution: Use a circle of radius d to find a point on AC that has the same distance to C as B.

Construction: Draw a circle with radius d around A and mark the intersection with AC as F.

Draw a line through F and B and mark the midpoint of FB as P.

Draw an orthogonal to FB through P and mark the intersection of this line with AC as C.

11. Number of Problem: 36

Difficulty: 1.8

Problem: Given a point P and two parallel lines, draw the (red) circle through P which is tangent to both lines.

Short solution: Use a third parallel and the fact that the center of the circle as the distance of r to the point P

Construction: Draw an orthogonal to a point (I_1) on one parallel and mark the intersection with the other parallel as I_2 .

Mark the midpoint of I_1 and I_2 as T and draw the orthogonal to I_1I_2 through T .

Draw a circle around P with radius $TI_2=r$ and mark the intersection with the circle and the third parallel as M .

Draw a circle around M through P .

12. Number of Problem: 40

Difficulty: 1.5

Problem: Draw the (red) circle which is tangent to the two given parallel lines and externally tangent to a given circle.

Short solution: Use third parallel and create circle around L with radius $r+r$ (tangent circle) to find center of circle

Construction: Draw an orthogonal to a point I_1 on one parallel and mark the intersection with the other parallel as I_2 .

Mark the midpoint of I_1 and I_2 as Z and draw a third parallel through Z .

Pick a point P on the tangent circle and draw a line through P and L .

Draw a circle around P with radius $r = \text{length of } I_1Z$ and mark the intersection with the line through L and P with O .

Draw a circle around L with the radius = length of LO and mark the intersection with the added parallel with M .

Draw a circle around M with the radius = length of I_1Z .

13. Number of Problem: 41

Difficulty: 1.8

Problem: Draw the (red) circle which is tangent to the two given parallel lines and internally tangent to a given circle.

Short solution: Use third parallel and create circle around L with radius $r+r$ (tangent circle) to find center of circle

Construction: Draw an orthogonal to a point I1 on one parallel and mark the intersection with the other parallel as I2.

Mark the midpoint of I1 and I2 as Z and draw a third parallel through Z.

Pick a point P on the tangent circle and draw a line through P and L.

Draw a circle around P with radius $r =$ length of I1Z and mark the intersection with the line through L and P that is closest to L with O.

Draw a circle around L with the radius $=$ length of LO and mark the intersection with the added parallel with M.

14. Number of Problem: 140

Difficulty: 1.1

Problem: Given a point P, and a line r , draw the mirror point of P about r

Short solution: Use two circles to get the mirror point.

Construction: Draw a circle around a point I1 on the line through P and mark the intersection of the circle with the line with I2.

Draw a circle around I2 through P and mark the intersection of the two circles as K. This is the mirror point.

15. Number of Problem: 143

Difficulty: 1.4

Problem: Given points A and B, find point C such that B is the midpoint of segment AC

Short solution: Use 4 circle to get reflect the point A on B.

Construction: Draw a circle around A through B and around B through A and mark an intersection with I1.

Draw a circle around I1 through B and mark the intersection with the circle around B with I2.

Draw a circle around I2 through B and mark the new intersection with the circle around B with C.

Image:

16. Number of Problem: 152

Difficulty: 1.2

Problem: Find the centre of the given circle

Short solution: Create two lines that are going through the center and mark the intersection.

Construction: Draw a circle around a point I1 on the circle and mark an intersection of this circle with the original circle with I2.

Draw a circle around I2 through I1 and draw a line through the points of intersections of the two circles that is called i

Repeat this with another point K1 on the circle to find the line k.

Mark the intersection of i and k with M. This is the center of the circle.

17. Number of Problem: 537

Difficulty: 1.3

Problem: Book III Prop. XVII : From a given point to draw a straight line touching a given circle (a tangent line).

Short solution: Use the circle of tales

Construction: To create the circle of tales, find the Midpoint of P and the Centrum of the circle(M), by drawing a circle through P around M and through M around P and draw a Line through the intersections. The intersection of this line with the line through M and P is the Midpoint of M and P.(called I) Draw a circle around I through P. The intersection of this circle with the original circle around M is the tangent point K. The line through K and P is the tangent.

18. Number of Problem: 548

Difficulty: 1.3

Problem: Book IV Prop. VIII: To inscribe a circle in a given square

Short solution: Find Midpoint of AB and the center of the square.

Construction: Draw a circle around A through B and around B through A and connect the intersections. The intersection of this line and AB mark as K. Draw a line through B and D and mark the intersection with the perpendicular bisector of AB as M. Its the Midpoint of the circle. Draw a circle around M through K.

19. Number of Problem: 549

Difficulty: 1.1

Problem: Book IV Prop. IX: To circumscribe a circle about a given square.

Short solution: Find the midpoint of the square and draw the circle.

Construction: Draw a line through A and C and a line through B and D and mark the intersection with M.

Draw a circle around M through A.

20. Number of Problem: 1492

Difficulty: 1.4

Problem: Draw the polar line of a point P with respect to a given circle

Short solution: Find the tangents to the circle through P and connect the tangent points on the circle to find the polar.

Construction: Draw a circle around M through P and around P through M and mark the intersections of the circles with I1 and I2.

Draw a line through I1 and I2 and mark the intersection with MP with K.

Draw a circle around K through P and mark the intersections with the original circle around M with T1 and T2.

Draw a line through T1 and T2 do get the polar.