Page	Line	Where is	Should be
3	19	V	W
14	-2	a	p
17	4	$(f^*(a)$	$(f^*(a))$
17	-7	$)\mathcal{U}(F)) = \phi_L^{-1}[U(F)]$	$(\mathcal{U}(F)) = \phi_L^{-1}[\mathcal{U}(F)]$
18	8	σ_X	σ_L
18	-6	$\neq$	$=$
18	-1	$\Sigma'_b \not\leq \Sigma'_a$	$\Sigma'_b \not\subseteq \Sigma'_a$
20	14	X	x
21	4	characteristics	characteristic
21	-9	$\uparrow x \in U$	$\uparrow x \subseteq U$
22	4	if and only if	if
22	-2	$\downarrow x \in U$	$J \in \tilde{U}$
26	13	right adjoint	left adjoint
33	13	$x \wedge x$	$x \wedge a$
39	13	the right adjoint of f^*	
39	-2	$f[S]$	$f[L]$
49	8	than	then
52	16	$\mathfrak{D}M$	$\mathfrak{D}S$
52	-10	$\bigvee_{x\in X} f(x)$	$\bigvee_{x\in S} f(x)$
53	14	$0 \leq X$	$\emptyset \geq X$
53	15	$X \cap Y$	$X \cup Y$
62		$\mathbf{2}$	Z
62	-6	$\Omega(q_i f) = f_i = h\phi^{-1}\phi\iota_i = h\iota_i = \Omega(f_i)$	$\Omega(q_i \cdot f) = h \cdot \phi^{-1} \cdot \phi \cdot \iota_i = h \cdot \iota_i = \Omega(f_i)$
79	-3	We have that	For any sober space X, we have that
90	16	$h\colon M \to L$	$h\colon L \to M$
94		proof of V.6.4.1(b)	See corrected proof below
126		proof of Lemma VII.1.4	See corrected proof below
128	-12	V.4.8	V.5.8
131	-14	$x = \{a \wedge x \mid a \in F\}$	$x = \bigvee\{a \wedge x \mid a \in F\}$
133	7	$\supseteq$	$\subseteq$
133	10	$v\sigma(a)$	$v\sigma(a) = a$
137	-14	if $D \subseteq L$	if $D \subseteq L$ is directed
147	-12	$\mathfrak{U}$	$\mathfrak{U}^\circ$
158	8	$\bigcup$	$\bigvee$
158	8	$\mathfrak{o}(U)$	$\mathfrak{o}(u)$
217	-5	Bring	Bing
232	-8	III.7.3	III.6.3
232	-7	$(h \oplus h)_{-1}(\mathfrak{o}(E)) = \mathfrak{o}((h \oplus h)^*(E)) \in \mathcal{E}$ for any $\mathfrak{o}(E) \in \mathcal{F}$	$((h \oplus h)_*)_{-1}(\mathfrak{o}(E)) = \mathfrak{o}((h \oplus h)(E)) \in \mathfrak{o}(\mathcal{F})$ for any $E \in \mathcal{E}$
293	5	$f(p, q) = 1$	$\mathbf{p} \leq f \leq \mathbf{q}$
300	7	$\gamma\varepsilon$	$\varepsilon\gamma$
302	-11	$h'(x) = x$	$h'(x) = h(x)$
302	-1	$\uparrow c \oplus \uparrow c = \uparrow(c \oplus c)$	$\uparrow c \oplus \uparrow c = \uparrow((c \oplus 1) \vee (1 \oplus c))$
303	2	$\overline{\mu}(x) = \mu(x)$	$\overline{\mu}(x) = (\overline{h} \oplus \overline{h})\mu(x)$

Page	Line	Where is	Should be
303	11	$h'\varphi = h'\varphi$ resp. $(h'\oplus h')\varphi = (h'\oplus h')\varphi$	$h'\varphi = h'\psi$ resp. $(h'\oplus h')\varphi = (h'\oplus h')\psi$
306	15	$v \nleq u_1$	$v \not\prec u_1$
307	-10	$\bigvee\{x \mid \exists y \neq 0, x \oplus y \leq u\}$	$\bigwedge\{x \mid u \leq x \oplus 1\}$
309	3	$((\mu \oplus \mathrm{id})\mu)$	$((\mu \oplus \mathrm{id})\mu)_{\#}$
311	-7	common refinement $\mathcal{E} \wedge \mathcal{F}$	supremum $\mathcal{E} \vee \mathcal{F}$
319	8	$(h(f(y)) = y$	$f(h(y)) = y$
319	9	$h \geq f$	$h \geq g$
321	18	least	greatest
321	19	greatest	least
321	-7	$X \smallsetminus g[Y \smallsetminus f[X \smallsetminus M]]$	$X \smallsetminus g[Y \smallsetminus f[M]]$
321	-5	$g[Y \smallsetminus f[X \smallsetminus A]]$	$g[Y \smallsetminus f[A]]$
331	12	lattice	bounded lattice
331	16	lattice	bounded lattice
331	-7	semilatice	semilattice
332	-9	lattice	bounded lattice
333	11	lattice	bounded lattice
344	-8	suprema (resp. infima)	infima (resp. suprema)

- **(Mono)coreflectivity of FitFrm in Frm** (pp. 94-95):

The proof used for coreflectivity of fitness (V.6.4.1(b)) is confused and incorrect (luckily enough, it was not used anywhere else in the book). Here is a better one:

1. Let $f\colon L \to M$ denote the localic map associated with a frame homomorphism $h\colon L \to M$. Recall that for the image and coimage of a sublocale (III.4) one has $f[S] \subseteq T$ iff $S \subseteq f_{-1}[T]$ and hence $f_{-1}[-]$ preserves meets. Recall the formulas (III.6.3)

$$f_{-1}[\mathfrak{c}(a)] = \mathfrak{c}(h(a)) \quad \text{and} \quad f_{-1}[\mathfrak{o}(a)] = \mathfrak{o}(h(a)). \tag{1.1}$$

2. For a frame L define

$$F_1(L) = \bigcap\{\mathfrak{o}(x) \mid \mathfrak{c}(a) \subseteq \mathfrak{o}(x)\}\}.$$

Explicitly, $a \in F_1(L)$ iff

$$(a \vee u = 1 \Rightarrow u \to x = x) \Rightarrow x \geq a. \tag{$*$}$$

Lemma. *$F_1(L)$ is a subframe of L, and $F_1(L) = L$ iff L is fit.*

Proof. Obviously, $0, 1 \in F_1(L)$. Let $a_i \in F_1(L)$. We will show that $\bigvee a_i$ satisfies $(*)$. Assume

$$\bigvee a_i \vee u = 1 \Rightarrow u \to x = x$$

and suppose that $a_i \vee u = 1$. Then $\bigvee a_i \vee u = 1$ and consequently $u \to x = x$. Since a_i satisfies $(*)$, we have that $x \geq a_i$ for every i and hence $x \geq \bigvee a_i$.

Finally let $a, b \in F_1(L)$. We have

$$\mathfrak{c}(a \wedge b) = \mathfrak{c}(a) \vee \mathfrak{c}(b) = \bigcap\{\mathfrak{o}(x) \mid \mathfrak{c}(a) \subseteq \mathfrak{o}(x)\} \vee \bigcap\{\mathfrak{o}(y) \mid \mathfrak{c}(b) \subseteq \mathfrak{o}(y)\} =$$
$$= \bigcap\{\mathfrak{o}(x \vee y) \mid \mathfrak{c}(a) \subseteq \mathfrak{o}(x), \mathfrak{c}(b) \subseteq \mathfrak{o}(y)\} \supseteq \bigcap\{\mathfrak{o}(u) \mid \mathfrak{c}(a) \vee \mathfrak{c}(b) \subseteq \mathfrak{o}(u)\}.$$

This shows that $F_1(L)$ is a subframe of L.

The second statement is the definition of fitness. □

3. For ordinals α define F_α as follows:

$$F_0(L) = L,\ F_{\alpha+1} = F_1(F_\alpha(L)) \text{ and } F_\alpha(L) = \bigcap_{\beta<\alpha} F_\beta(L) \text{ for a limit ordinal.}$$

Since $F_\alpha(L)$ decrease there is an ordinal $\gamma(L)$ such that $F_1(F_{\gamma(L)}(L)) = F_{\gamma(L)}(L)$. Set

$$F(L) = F_{\gamma(L)}(L).$$

4. Theorem. *F can be extended to a functor* **Frm** $\to$ **FitFrm** *and together with the inclusion homomorphisms $\iota_L\colon F(L) \to L$ it constitutes a coreflection.*

Proof. It suffices to show that for each frame homomorphism $h\colon L \to M$ one has

$$h[F_1(L)] \subseteq F_1(M).$$

Let $a \in F_1(L)$ and consider the localic map f adjoint to h. We have

$$\mathfrak{c}(a) = \bigcap\{\mathfrak{o}(x) \mid \mathfrak{c}(a) \subseteq \mathfrak{o}(x)\}$$

and hence, by (1.1) and since $f_{-1}[-]$ preserves meets,

$$\mathfrak{c}(h(a)) = f_{-1}[\mathfrak{c}(a)] = f_{-1}[\bigcap\{\mathfrak{o}(x) \mid \mathfrak{c}(a) \subseteq \mathfrak{o}(x)\}] = \bigcap\{f_{-1}[\mathfrak{o}(x)] \mid \mathfrak{c}(a) \subseteq \mathfrak{o}(x)\} =$$
$$= \bigcap\{\mathfrak{o}(h(x)) \mid \mathfrak{c}(a) \subseteq \mathfrak{o}(x)\} \supseteq \bigcap\{\mathfrak{o}(y) \mid \mathfrak{c}(h(a)) \subseteq \mathfrak{o}(y)\}. \quad \square$$

• **Corrected proof of Lemma VII.1.4** (page 126):

Proof. First, observe that if β is a limit ordinal and $(1, c) \in \pi_1\nu_\beta(U)$ then $(1, c) \in \nu_\gamma(U)$ for some $\gamma < \beta$: indeed, by compactness there is a finite A with $\bigvee A = 1$ and $A \times \{c\} \subseteq \bigcup\{\nu_\gamma(U) \mid \gamma \text{ non-limit},\ \gamma < \beta\}$ and by IV.5.6 then $(1, c) \in \nu_\gamma(U)$.

Now let the statement not hold. Then there exists $(1, b) \in \nu_\alpha(U)$ with $\alpha > 1$ least among such $(1, b)$'s. Obviously α is not a limit ordinal, and by the observation α is not a successor of a limit ordinal β either (else $b = \bigvee B$ such that all the $c \in B$ are in a $\nu_\gamma(U)$ and by the minimality of α, $(1, c) \in \pi_2\pi_1(U)$ and $(1, b) \in \pi_2\pi_2\pi_1(U) = \pi_2\pi_1(U)$, a contradiction). Thus, $(1, b) \in \nu\nu(V) = \pi_2\pi_1\pi_2\pi_1(V)$ for $V = \nu_\gamma(U)$. Then $b = \bigvee B$ for some B with $\{1\} \times B \subseteq \pi_1\pi_2\pi_1(V)$, and for each $c \in B$, $(1, c) \in \pi_1\pi_2\pi_1(V)$ and we have A_y such that $\bigvee A_y = 1$ and $A_y \times \{y\} \subseteq \pi_2\pi_1(V)$. By compactness we can assume that A_y is finite and by IV.5.6, $(1, y) = (\bigvee A_y, y) \in \pi_2\pi_1(V)$ and hence $(1, b) = (1, \bigvee B) \in \pi_2\pi_2\pi_1(V) = \pi_2\pi_1(V) = \nu_{\alpha-1}(U)$, a contradiction. $\square$

• **Corrected proof of Proposition XV.5.2.2** (page 311):

The last four lines of the proof of Proposition 5.2.2 in Chapter XV are incorrect. When mending the error we found that it can also be made more transparent. It goes as follows.

Proof. We will show that

$$\mathcal{E} = \{E \mid E \text{ entourage, } E \geq E(a), a \in N\} =$$
$$= \mathcal{E}_\mathcal{U} = \{E \mid E \text{ entourage, } E \geq E_{U(a)}, a \in N\}.$$

We have $E_{U(a)}(= \bigvee\{x \oplus x \mid x \oplus \gamma(x) \leq \mu(a)\}) \leq E(a)$.

To obtain an estimate from the other side, choose by 4.2.4 $b, c \in N$ such that $b * b^{-1} \leq c$ and $c * c^{-1} \leq a$. Let $x \oplus y \leq E(b)$. We can assume $x \oplus y \neq 0$, hence $x \neq 0 \neq y$. First, as $y \neq 0$, we have by 4.2.3 ((2), (3) and (5)) that $y^{-1} * y \in N$ and further

$$x*x^{-1} \leq (x*(y^{-1}*y))*x^{-1} = (x*y^{-1})*(y*x^{-1}) \leq b*b^{-1} \leq c \quad \text{and} \quad x*y^{-1} \leq b \leq b*b^{-1} \leq c$$

and hence $(x, x), (x, y) \in E(c)$ and since $E(c)$ is saturated we have, for $z = x \vee y$, $(x, z) \in E(c)$, that is, $x * z^{-1} \leq c$.

Similarly, we have also $y * x^{-1} \leq b^{-1} \leq b * b^{-1} \leq c$, hence also $(z, x) \in E(c)$, that is, $z * x^{-1} \leq c$. Then, as $x \neq 0$, we have by 4.2.3 ((2), (3) and (5)) again,

$$z * z^{-1} \leq (z * (x^{-1} * x)) * z^{-1} \leq z * x^{-1} * x * z^{-1} \leq c * c^{-1} \leq a$$

so that $x \oplus y \leq z \oplus z \leq E_{U(a)}$. Thus, $E(b) \leq E_{U(a)}$. □