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New aspects of subfitness in frames and spaces. (English) Zbl 06641512

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A locale is subfit if each of its open sublocales is a join (in the coframe of sublocales) of closed sublocales. This localic separation axiom was introduced by *J. R. Isbell* [Math. Scand. 31, 5–32 (1972; Zbl 0246.54028)]. A topological space is symmetric if every open subset is the union of closed ones. But for the fact that joins in the coframe of sublocales are not unions, in general, these definitions seem identical. This resemblance notwithstanding, there are subfit spaces which are not symmetric. One of the aims of this paper is to explain this seeming discrepancy. The authors trace this to what they aptly call "an imperfect representation of subspaces as sublocales in non-T_D spaces".

To say a frame is spatial is to say it is isomorphic to the frame of open sets of some space, without saying anything about that space. As is well known, a frame is spatial if and only if each of its elements is a meet of prime elements. Replacing primes by maximal elements, the authors define a frame to be T_1-spatial if every element is a meet of maximal elements. The appropriateness of the name is fully justified by the result that a frame is T_1-spatial precisely when it is isomorphic to the frame of open sets of a T_1-space. T_1-spatiality is shown to be the conjunction of subfitness with T_D-spatiality (meaning: isomorphic to the frame of open sets of a T_D-space), and also to be the conjunction of subfitness with what the authors call set-boundedness.

In the penultimate section the authors give some other characterizations of subfitness. They call a sublocale (resp. subspace) replete if it has non-void intersection with every non-void sublocale (resp. subspace). For any $a \in L$, they define the sublocale $\mathfrak{so}(a)$ to be the join of open sublocales contained in the open sublocale $\mathfrak{o}(a)$. They then show that L is subfit if and only if the only replete sublocale of L is L itself, if and only if for every $a \in L$, $\mathfrak{so}(a) = \mathfrak{o}(a)$. Define a relation $\approx$ on the lattice of subsets of a topological space X by stipulating that $Y \approx Z$ in case for every pair of open sets U, V of X, $U \cap Y = V \cap Y$ iff $U \cap Z = V \cap Z$. With this, the authors show that a space X is subfit if and only if for every open set U, $U \approx \mathfrak{s}(U)$, where the latter denotes the union of all closed sets of X that are contained in U. This is indeed a beautiful result because it informally says X is subfit precisely when every open set is "approximately" equal to a union closed ones.

The final section is about the expressibility of the Heyting implication (and, in particular, the pseudocomplement) as a meet. For the Heyting implication in general, the frame must be subfit, and conversely. For the pseudocomplement, the frame must be weakly subfit (a proper weakening of fitness), and conversely.

Reviewer: Themba Dube (Unisa)

MSC:

06D22 Frames, locales
54B05 Subspaces (general topology)
54D10 Lower separation axioms (T_0–T_3, etc.)
54E15 Uniform structures and generalizations
54E17 Nearness spaces

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frame; locale; sublocale; sublocale lattice; subfitness; T_1-spatiality; T_D-spatiality; repleteness

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