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Semi-localizations of semi-abelian categories

A semi-localization of a category is a full reflective subcategory with the property that the reflector is a semi-left-exact functor. In this article we first determine an abstract characterization of the categories which are semi-localizations of an exact Mal'tsev category, by particularizing a result due to S. Mantovani [1]. We then turn our attention to semi-abelian categories, where a special type of semi-localizations are known to correspond to torsion theories [2]. For this purpose a new characterization of protomodular categories is obtained, on the model of the one discovered by Z. Janelidze in the pointed context [3]. Both the torsion-free and the hereditarily-torsion-free subcategories of semi-abelian categories are then characterized, and some examples are examined in detail. We finally explain how these results extend similar ones obtained by W. Rump in the abelian context [4].

References:

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- [2] D. Bourn and M. Gran, Torsion theories in homological categories, *J. Algebra* 305 (2006) 18–47.
- [3] Z. Janelidze, Closedness Properties of Internal Relations III: Pointed Protomodular Categories, *Appl. Categ. Structures* 15 (3) (2007) 325–338.
- [4] W. Rump, Almost abelian categories, *Cahiers Topologie Géom. Différentielle Catég.*, 42 (3) (2001) 163–225.

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