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Push forwards of crossed squares

It is well known that given a crossed module $\partial : G_1 \to G_0$ of groups, then:

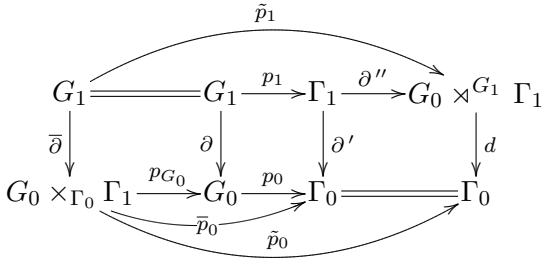
$\ker \partial$ is G_0-invariant, so that $\ker \partial \to G_0$ is a crossed module;

the action of G_0 on the abelian group $\ker \partial$ passes to $\operatorname{coker} \partial$ so that $\ker \partial \to \operatorname{coker} \partial$ is still a crossed module.

We show that there is a corresponding result if we start with a crossed square (an internal crossed module in the category of crossed modules):

$$\begin{array}{ccc} G_1 & \xrightarrow{p_1} & \Gamma_1 \\ {\scriptstyle\partial}\downarrow & & \downarrow{\scriptstyle\partial'} \\ G_0 & \xrightarrow{p_0} & \Gamma_0 \end{array}$$

and we take the homotopical version of kernels and cokernels, using pullbacks for the first and push forwards for the second, so that in the diagram



both $(p_1, \overline{p}_0)$ and $(\tilde{p}_1, \tilde{p}_0)$ give rise to crossed squares.

References:

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