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*Extending semilattices to frames*

Each (bounded) meet-semilattice  $S$  is well known to be freely extended to its downset frame  $\mathfrak{D}S$ . This extension, of course, does not respect the possible joins, and the question naturally arises when and how one can extend the semilattice to a frame preserving a given part of the existing join structure. Using the Johnstone's technique of coverages and sites, and a deep injectivity result by Bruns and Lakser one can show that the range of frame extensions of  $S$  is a sub-coframe (indeed an interval) of the coframe of the sublocales of  $\mathfrak{D}S$ , with the injective envelope of  $S$  as the bottom.

We will also briefly mention the relation of the extensions and the Dedekind-MacNeille completion, and a few further aspects of the construction involved.

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\*Joint work with R.N. Ball.