Pedro Resende
Instituto Superior Técnico, Lisboa

Étale groupoids and their quantales: functoriality

Both the C*-algebras of locally compact groupoids and the quantales of open localic groupoids are algebraic structures of "convolution type". Such constructions are not immediately functorial. For instance, on one extreme, the functor C from the category of compact Hausdorff spaces to the category of C*-algebras is contravariant, as is the open sets functor Ω from topological spaces to frames, whereas, on the other extreme, there is a covariant functor from discrete groups to C*-algebras, and, similarly, there is a covariant functor from discrete groups to quantales. In order to make sense of these extremes within a single definition one should use bicategories. This idea has appeared in several ways in the context of C*-algebras (see, e.g., [1, 2, 3]), and in this talk I explain how it applies to localic étale groupoids and their quantales. Indeed, here the situation is more satisfactory because one obtains an equivalence of bicategories, namely between the bicategory of localic étale groupoids with bi-actions as 1-cells, and the bicategory of inverse quantal frames, whose 1-cells are quantale bimodules. This is a natural functorial extension of the objects-only correspondence of [4].

References:

[1] M. Buneci, Groupoid categories, *Hot topics in operator theory*, 23–37, Theta Ser. Adv. Math., 9, Theta, Bucharest, 2008.

[2] J. Mrčun, Functoriality of the bimodule associated to a Hilsum–Skandalis map, *K-Theory* 18 (1999) 235–253.

[3] P.S. Muhly, J.N. Renault, D.P. Williams, Equivalence and isomorphism for groupoid C*-algebras, *J. Operator Theory* 17 (1987) 3–22.

[4] P. Resende, Étale groupoids and their quantales, *Adv. Math.* 208 (2007) 147–209.