

Thomas Weighill
Stellenbosch University

A characterisation of R_1 -spaces via approximate Mal'tsev operations

For an object X in a category \mathbb{C} , a morphism $\mu : X^3 \rightarrow A$ is an approximate Mal'tsev operation with approximation $\alpha : X \rightarrow A$ [2] if for any object C of \mathbb{C} and for any two morphisms $x, y : C \rightarrow X$ we have $\mu(x, y, y) = \alpha x = \mu(y, y, x)$. It was shown in [2] that a regular category [1] with coproducts is a Mal'tsev category [3] if and only if every object admits an approximate Mal'tsev co-operation whose approximation is a regular epimorphism. The dual of the category of topological spaces is regular, but not Mal'tsev, since not all topological spaces admit an approximate Mal'tsev operation whose approximation is a regular monomorphism (i.e. an embedding). In this talk we characterise those topological spaces which do. These spaces turn out to be precisely the R_1 -spaces [4] (also known in the literature as pre-regular spaces).

References:

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- [4] A. S. Davis, Indexed Systems of Neighborhoods for General Topological Spaces, *The American Mathematical Monthly* 68 (1961) 886–893.