A presentation of the book

Schreier split epimorphisms in monoids and in semirings

by D. Bourn, N. Martins-Ferreira, A. Montoli, and M. Sobral

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Universidade de Coimbra

Outline

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Introduction

During the last years there has been a great interest in finding a suitable categorical framework to study group-like structures :

- Mal'tsev categories
- protomodular categories
- homological categories
- semi-abelian categories

Some beautiful theories have been developed in these categories : commutators, homology, cohomology, torsion theories, radicals, etc.

These theories have led to a conceptual understanding of parallel results in Grp, Rng, Lie_K, XMod, Grp(Comp).

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Question

What can be said about the categorical properties of the category Mon of monoids ?

Although Mon is not a Mal'tsev category, it is a unital category (Bourn, 1996) :

Definition

A finitely complete pointed category $\mathcal{C}$ is unital when, given two objects A and B in $\mathcal{C}$, the morphisms $(1_A, 0)$ and $(0, 1_B)$ in the diagram

$$A \xrightarrow{(1_A,0)} A \times B \xleftarrow{(0,1_B)} B$$

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are jointly extremal epimorphic.

This means that, given a monomorphism $m\colon M \to A \times B$

$$\begin{array}{ccccc} & & M & & \\ & & \big\downarrow{\scriptstyle m} & & \\ A & \xrightarrow[(1_A,0)]{} & A \times B & \xleftarrow[(0,1_B)]{} & B \end{array}$$

such that $(1_A, 0)$ and $(0, 1_B)$ factor through m

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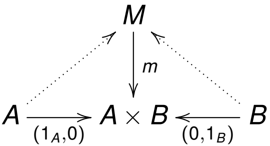

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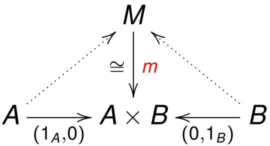

such that $(1_A, 0)$ and $(0, 1_B)$ factors through m, then m is an iso.

This implies in particular that the arrows

$$A \xrightarrow{(1_A,0)} A \times B \xleftarrow{(0,1_B)} B$$

are jointly epimorphic.

This opens the way to the study of commuting arrows :

given two arrows $a\colon A \to C$ and $b\colon B \to C$ with the same codomain, there is at most one arrow ϕ making the diagram

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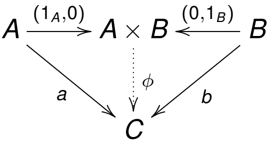

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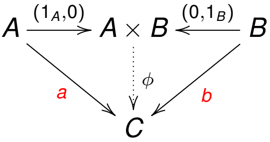

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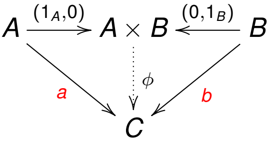

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Can one develop some other aspects of categorical algebra in Mon ?

Is there a structural property of the fibration of points in Mon, as it is the case in the category Grp of groups ?

The book Schreier split epimorphisms in monoids and in semirings gives a positive and very interesting answer !

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Recall that the fibration of points concerns the category $\mathrm{Pt}(\mathbb{C})$:

- objects : split epimorphisms in $\mathbb{C}$

$$A \underset{s}{\overset{p}{\rightleftarrows}} B \qquad\qquad ps = 1_B$$

- morphisms : pairs of arrows (f_A, f_B) in $\mathbb{C}$ making the diagram

$$\begin{array}{ccc} A & \underset{s}{\overset{p}{\rightleftarrows}} & B \\ {\scriptstyle f_A}\downarrow & & \downarrow{\scriptstyle f_B} \\ A' & \underset{s'}{\overset{p'}{\rightleftarrows}} & B' \end{array}$$

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commute.

There is a functor $P\colon \mathrm{Pt}(\mathbb{C}) \to \mathbb{C}$ associating, with any split epimorphism, its codomain :

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One discovery in this book is that, in Mon, one should consider SPt(Mon), the category of "Schreier split epimorphisms in Mon" :

let

$$0 \longrightarrow K \xrightarrow{k} A \underset{s}{\overset{p}{\rightleftarrows}} B \longrightarrow 0$$

be a split epi in Mon, with kernel $k\colon K \to A$.

This is a Schreier split epi if, for any $a \in A$, there is a unique $k \in K$ such that

$$a = k \cdot sp(a).$$

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Remark

Any Schreier split epi in Mon determines a set-theoretic map q

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defined by $q(a) = k$, for any $a \in A$, where $k \in K$ is such that

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Example

The canonical split epi in Mon given by

$$0 \longrightarrow A \underset{(1_A,0)}{\overset{\pi_A}{\leftrightarrows}} A \times B \underset{(0,1_B)}{\overset{\pi_2}{\rightleftarrows}} B \longrightarrow 0$$

is a Schreier split epi.

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Any split epimorphism

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in the category Grp is a Schreier split epi :

indeed, given $a \in A$, choose $q(a) = k = a \cdot sp(a)^{-1} \in K$, and

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In the category Mon, the Schreier split epis behave extremely well :

Lemma
Given a Schreier split epimorphism in Mon equipped with its kernel

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then $p = \mathrm{coker}(k)$:

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Theorem

Given a commutative diagram of Schreier split exact sequences

$$
\begin{array}{ccccccccc}
0 & \longrightarrow & K & \xrightarrow{k} & A & \underset{s}{\overset{p}{\rightleftarrows}} & B & \longrightarrow & 0 \\
 & & {\scriptstyle u}\downarrow & & \downarrow{\scriptstyle v} & & \| & & \\
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\end{array}
$$

in Mon, if u is an iso then v is an iso.

An analogy then appears between the situations in Grp and in Mon :

Groups

For any $f\colon X \to Y$ in Grp the change-of-base functor

$$f^*\colon \mathrm{Pt}_Y(\mathrm{Grp}) \to \mathrm{Pt}_X(\mathrm{Grp})$$

with respect to the fibration $P\colon \mathrm{Pt}(\mathrm{Grp}) \to \mathrm{Grp}$ is conservative.

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For any $f\colon X \to Y$ in Mon the change-of-base functor

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The full subcategory SPt(Mon) of Pt(Mon) determines a subfibration P^S of the fibration of points P :

$$\begin{array}{ccc} \mathrm{SPt(Mon)} & \overset{j}{\rightarrowtail} & \mathrm{Pt(Mon)} \\ & {\scriptstyle P^S} \searrow \quad \swarrow {\scriptstyle P} & \\ & \mathrm{Mon} & \end{array}$$

These observations lead to a detailed study of internal categorical structures in Mon :

- Schreier internal categories (Patchkoria, 1998),
- Schreier internal groupoids,
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Split extension classifier

In Mon, for any monoid M, it is shown that the monoid $\mathrm{End}(M)$ of endomorphisms of M has a universal property, which is analogous to the one of the automorphism group $\mathrm{Aut}(G)$ of a group G in Grp.

Indeed, one can construct a Schreier split extension

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for any Schreier split extension with kernel M in Mon

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there is a unique arrow ϕ making the following diagram commute :

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For this reason the monoid $End(M)$ is called the Schreier split extension classifier of M .

The group $Aut(M)$ is also shown to have a universal property, and it is called the homogeneous split extension classifier of M.

These concepts are then used in order to classify what the authors call special Schreier extensions with abelian kernels.

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Semirings

Many of the interesting results discovered by Manuela Sobral and her collaborators in Mon also have analogous versions in the category SRng of semirings.

Definition

$(A, +, \cdot, 0)$ is a semiring if

- $(A, +, 0)$ is a commutative monoid ;
- $\cdot : A \times A \to A$ is an associative binary operation such that

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

$$(a + b) \cdot c = a \cdot c + b \cdot c.$$

Fact :

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$$a = k + sp(a).$$

The fibration

$$\mathsf{SPt}(\mathsf{SemiRng}) \to \mathsf{SemiRng}$$

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The book

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Texts in Mathematics of the Department of Mathematics of the University of Coimbra

sheds some new light on the categories Mon and SemiRng, by providing a categorical foundation to the study of monoids and semirings.

Happy Birthday Manuela !