A categorical model for 2-PDAs with states

Jürgen Koslowski

Department of Theoretical Computer Science
Technical University Braunschweig

cmat14, Coimbra

(2014-01-25)

http://www.iti.cs.tu-bs.de/~koslowj/RESEARCH

Content

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- We provide an elegant categorification of ss2PDAs that allows states to be incorporated orthogonally to storage. The result bears strong resemblance to the tile model of Gadducci and Montanari [2000] for rewriting and abstract concurrent semantics.

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- Joyal, Winskel and Nielsen [1994] as well as Cockett and Spooner [1997] approach bisimulations synthetically; in an enriched context this has been done by Schmitt and Worytkiewicz [2006].

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- Instead, one has to use modules rather than oplax natural transformations, from, resp., into the discrete lax functor $\Sigma^\star \xrightarrow{D} rel$. In the context of graphs this means that instead of Σ we need to consider the reflexive graph Σ^ϵ with hom-set $\Sigma + \{\epsilon\}$.

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- Walters wanted to illustrate his construction of the free category with products over a multi-graph. However, a more direct way of extracting the generated language becomes available with top-down parsing, hence we revert to co-multi-graphs or cm-graphs, for short.

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or simply as $X \xrightarrow{a} Y_0 \dots Y_{n-1}$, since γ is faithful.

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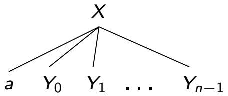

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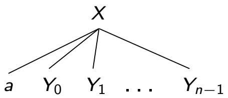

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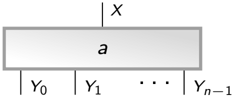

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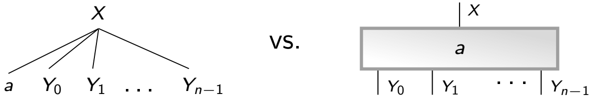

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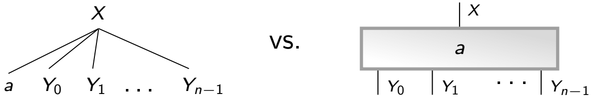

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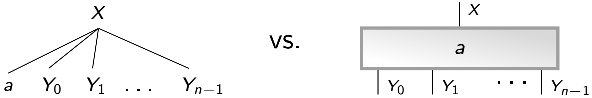

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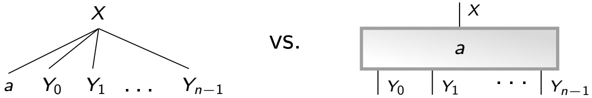

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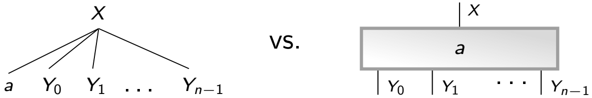

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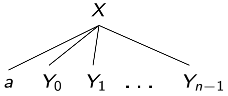

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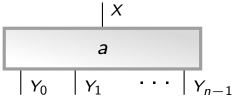

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- As terminals are not limited to leaves, we need to switch from positional ordering of trees to temporal ordering (rotation by $\pi/2$ indicates this), which requires some notion of current position.

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- Left and right moves $AB \xrightarrow{\epsilon} \epsilon AB$ and $AB \xrightarrow{\epsilon} AB\epsilon$ just change the current position.

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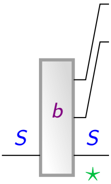

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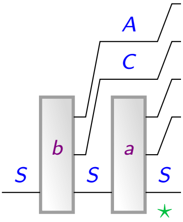

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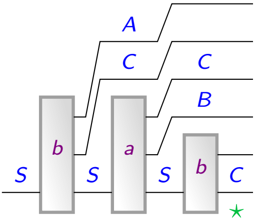

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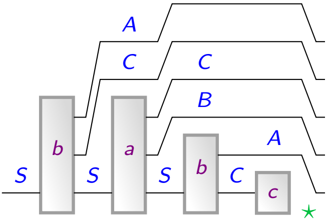

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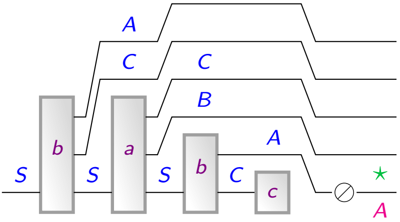

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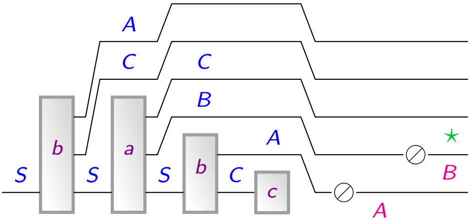

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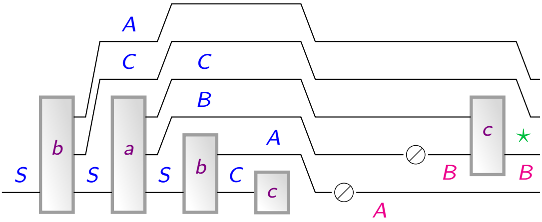

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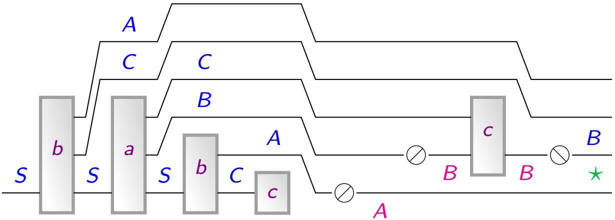

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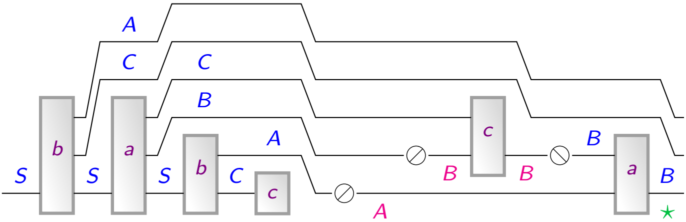

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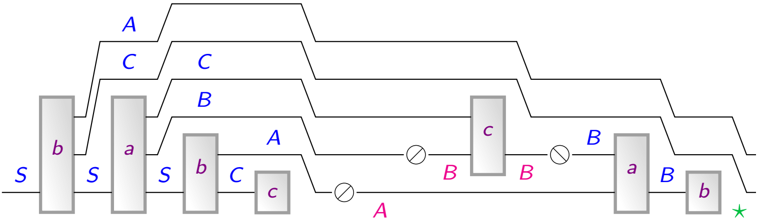

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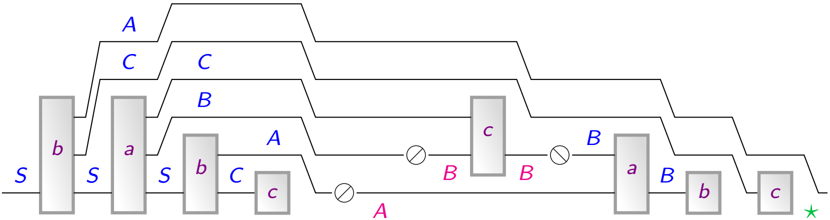

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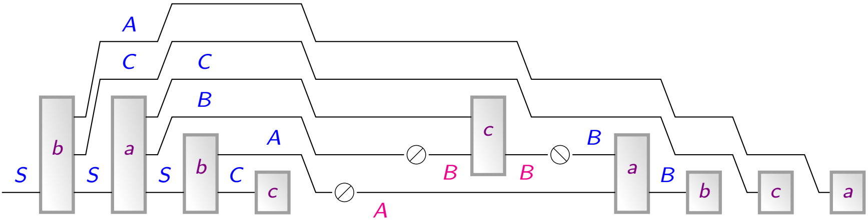

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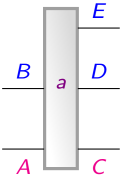

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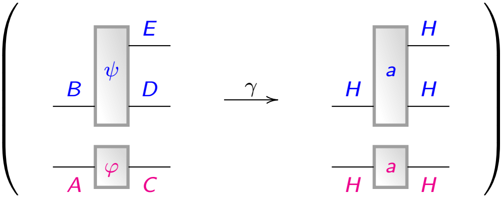

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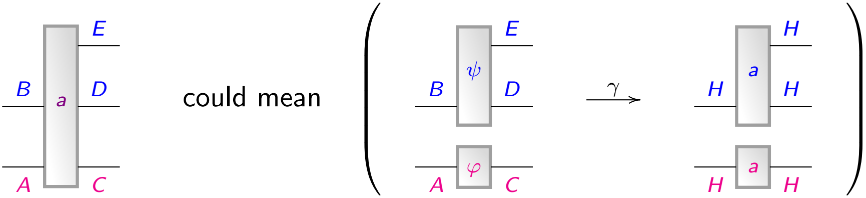

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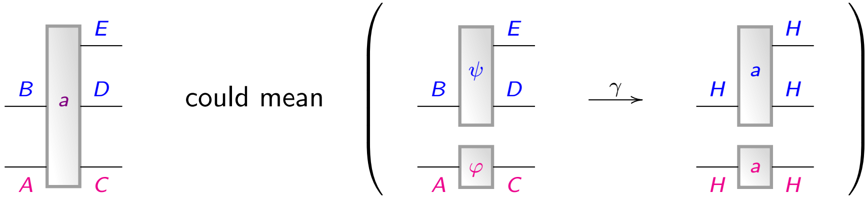

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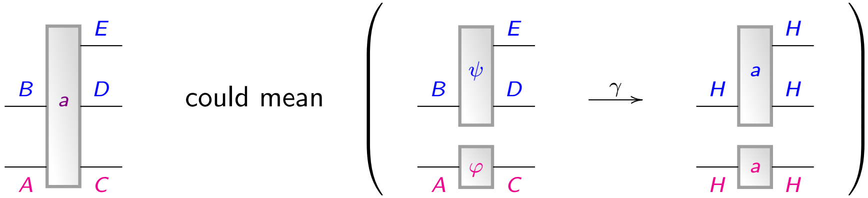

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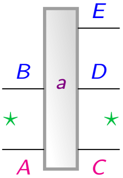

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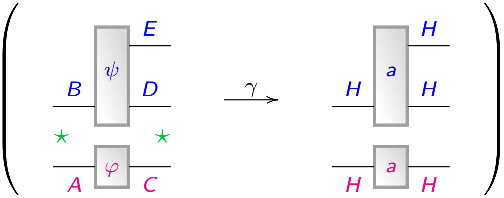

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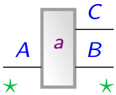

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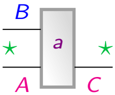

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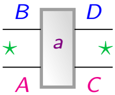

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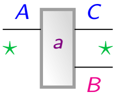

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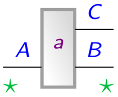

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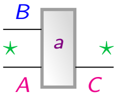

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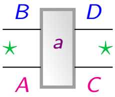

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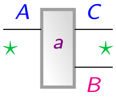

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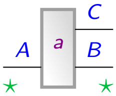

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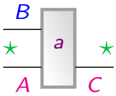

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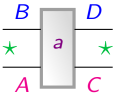

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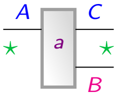

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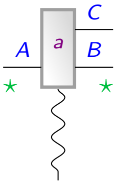

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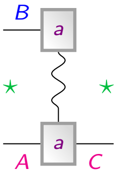

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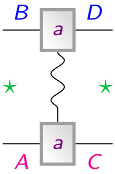

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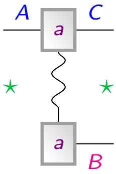

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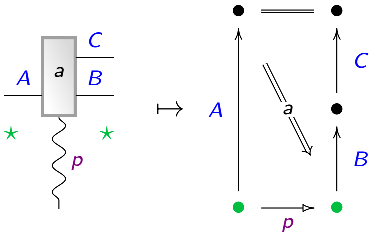

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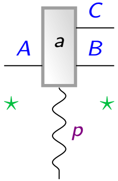

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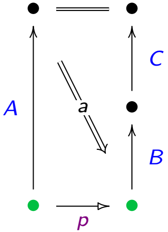

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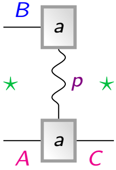

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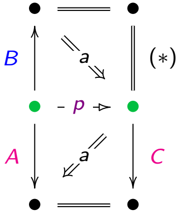

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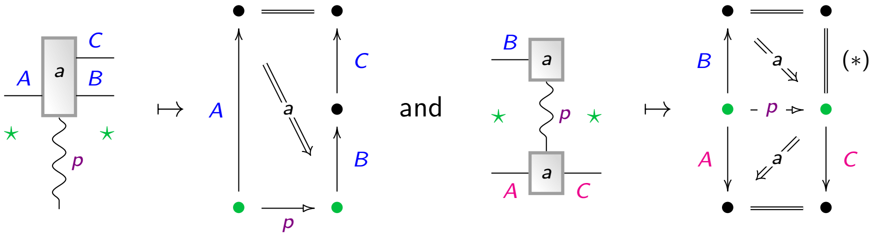

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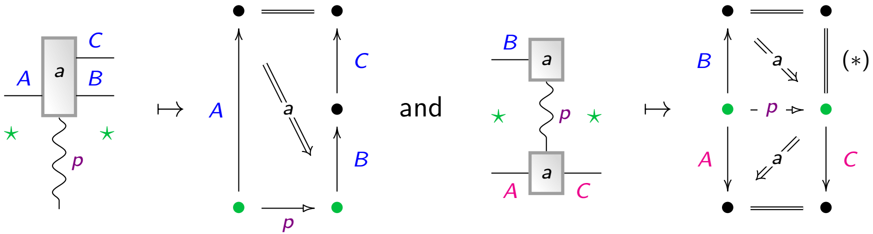

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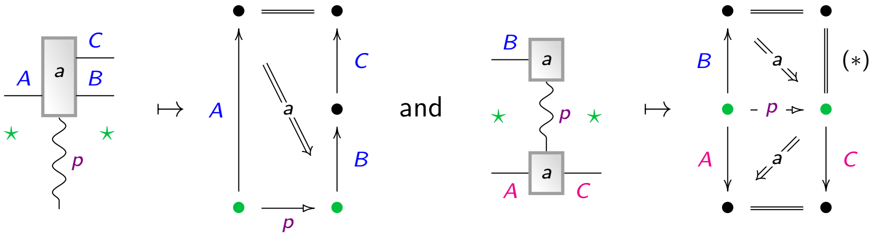

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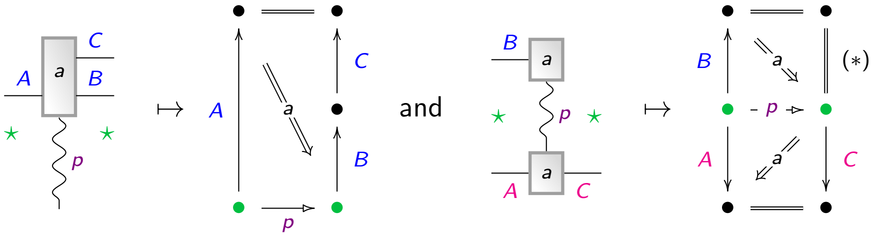

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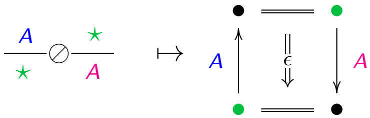

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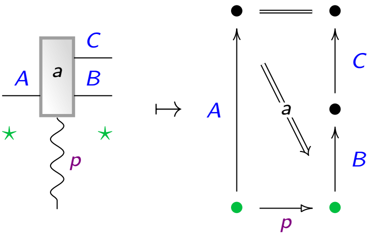

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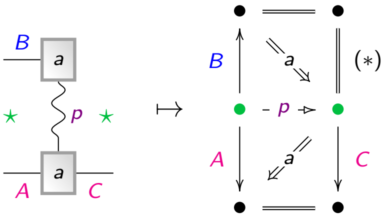

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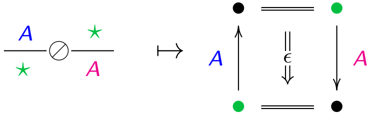

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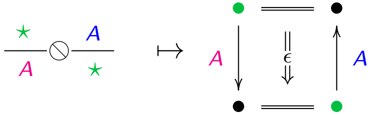

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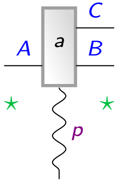

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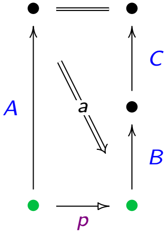

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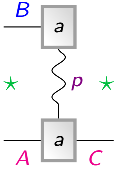

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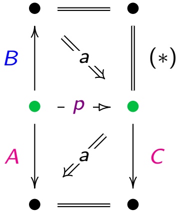

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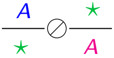

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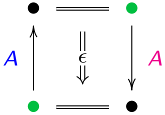

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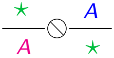

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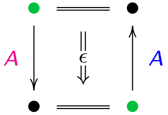

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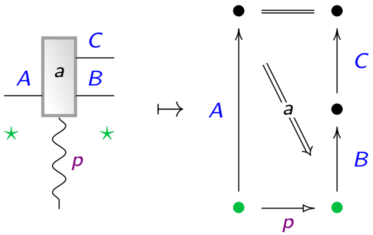

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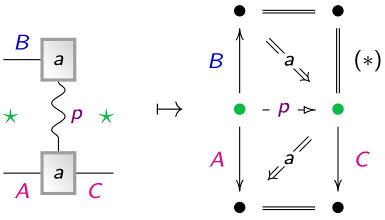

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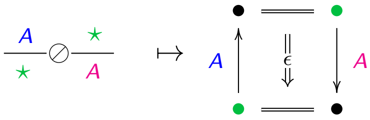

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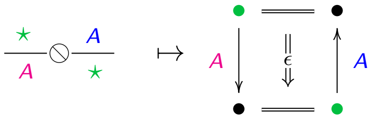

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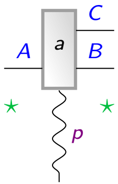

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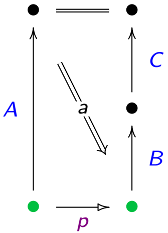

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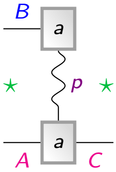

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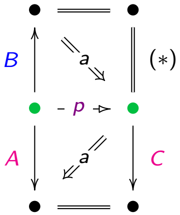

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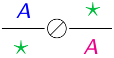

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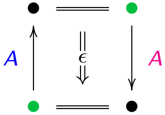

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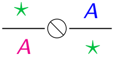

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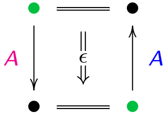

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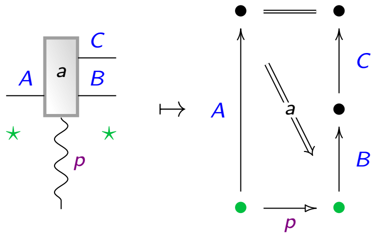

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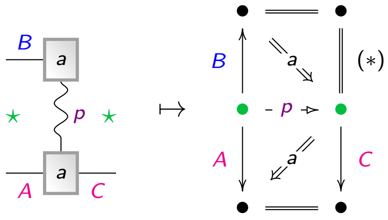

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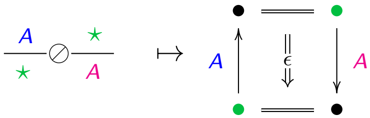

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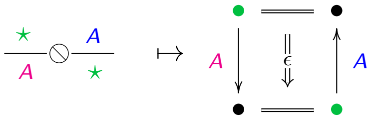

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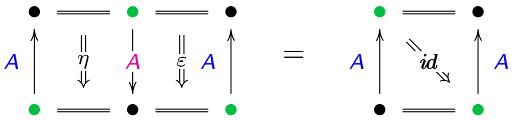

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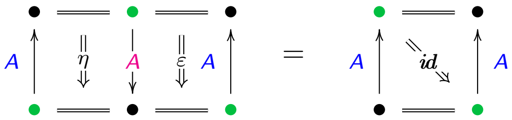

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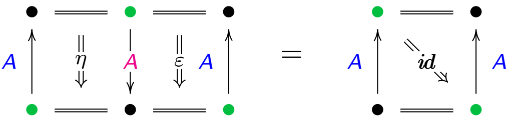

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