Approximating lengths of reset words

Mikhail V. Berlinkov

Ural State University, Ekaterinburg, Russia
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Synchronizing Automata

Let $\mathscr{A}$ be a complete **deterministic finite automaton** whose input alphabet is Σ and whose state set is Q.

- Denote by $S.v$ the image of the subset $S \subseteq Q$ under the action of the word $v \in \Sigma^*$.
- A word v is called reset (or synchronizing) word for $\mathscr{A}$ iff $|Q.v| = 1$ (equivalently $q.v = p.v$ for all $q, p \in Q$).
- $\mathscr{A}$ is called synchronizing if it possesses some reset word.
- $\mathfrak{C}(\mathscr{A})$ denotes the **minimum length** of reset words for $\mathscr{A}$ and this function is usually called Cerny function and let us call its value reset length of $\mathscr{A}$.

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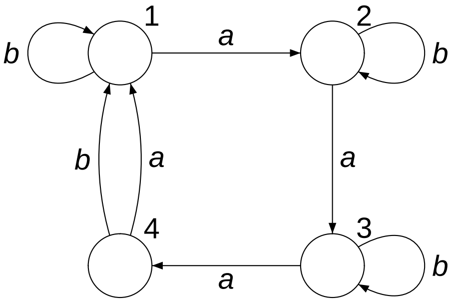
Synchronizing Automaton $\mathscr{A}$ by "greedy" algorithm



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Since $|Q.v| = 1$ the word v is a reset word for $\mathscr{A}$ whence $\mathfrak{C}(\mathscr{A}) \leq |v| = 10$.

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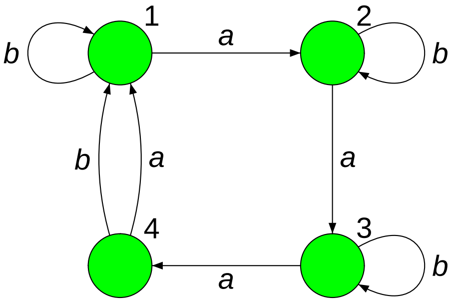

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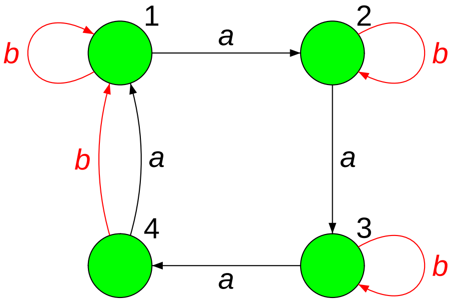

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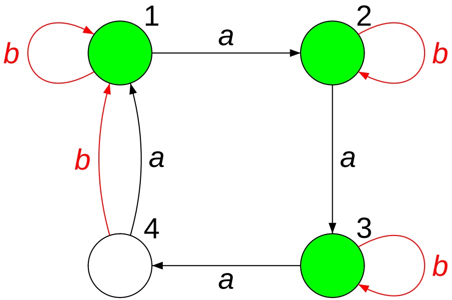

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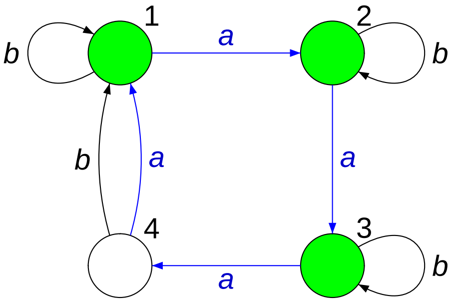

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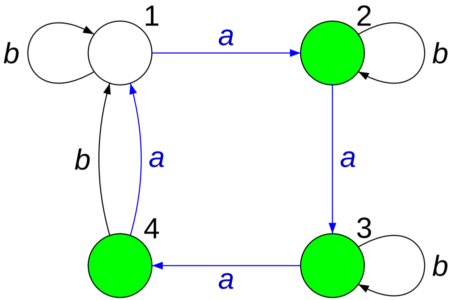

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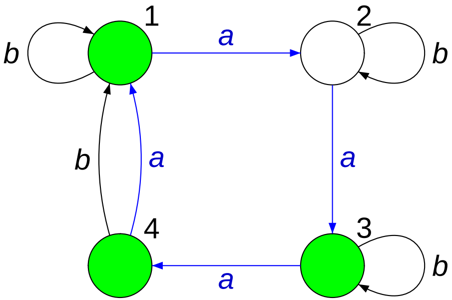

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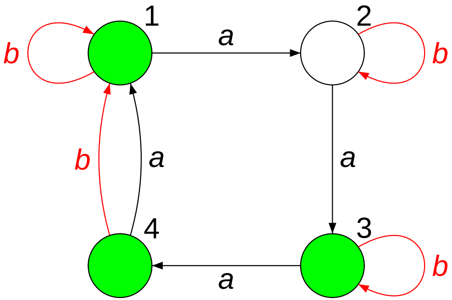

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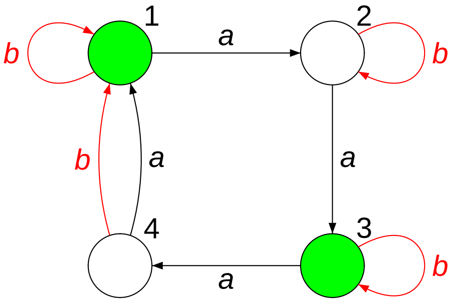

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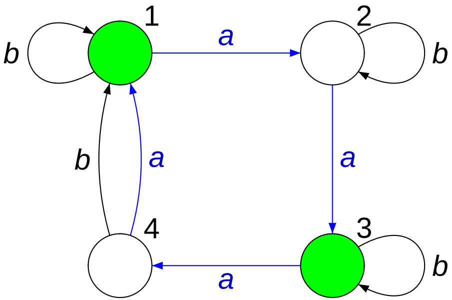

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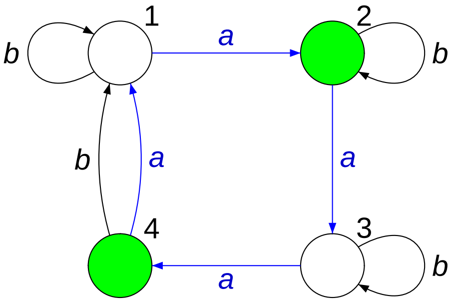

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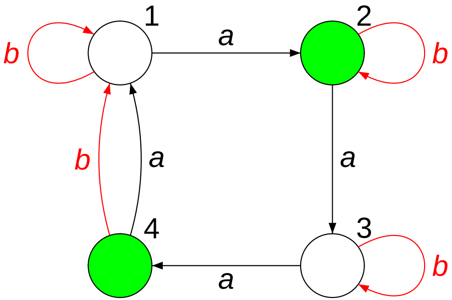

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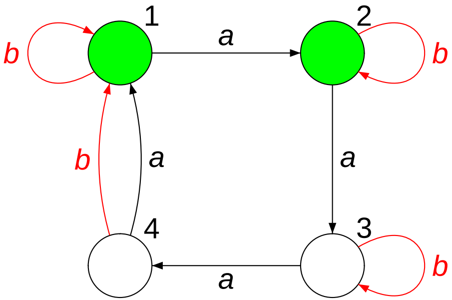

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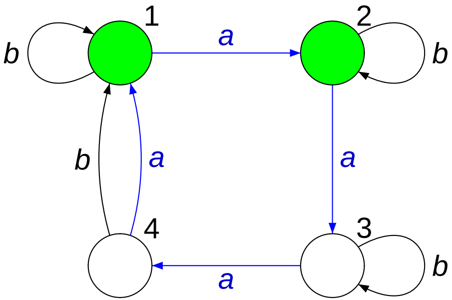

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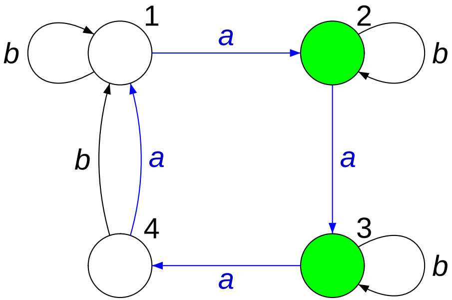

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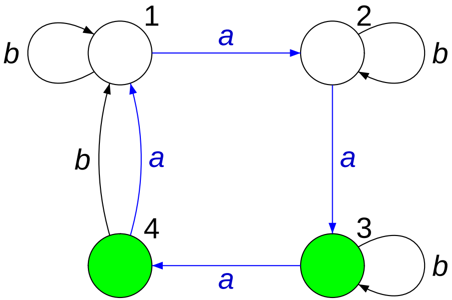

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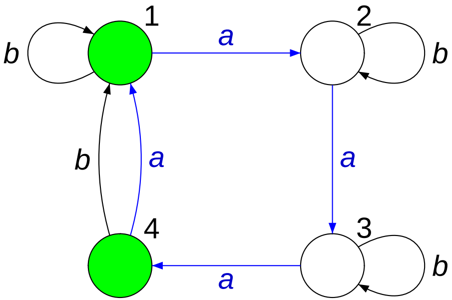

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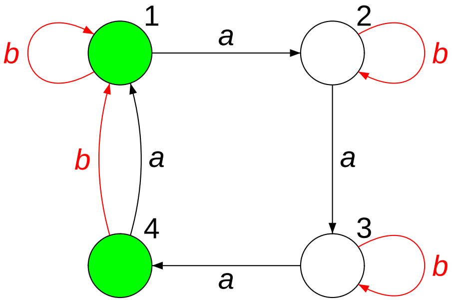

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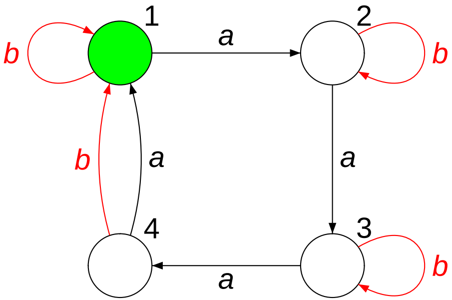

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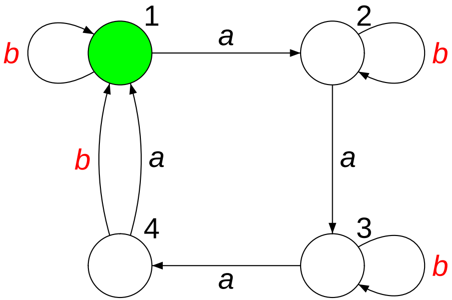

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Given an automaton $\mathscr{A}$;

- How to find some reset word for $\mathscr{A}$ if it exists?

Given a synchronizing automaton $\mathscr{A}$;

- How to find "relatively" short reset word for $\mathscr{A}$ or its length?

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Search For Some Reset Word

Synchronization Criterion | Černý, 1964

An automaton $\mathscr{A}$ is synchronizing iff each pair of states p, q can be merged by some word v, i.e. $p.v = q.v$.

Find-Sync-Word | in $O(n^3)$ (Greedy algorithm)

Given An n-state automaton $\mathscr{A}$;
Return Some reset word for $\mathscr{A}$ if it exists.

Check-Sync | in $O(n^2)$

Given An n-state automaton $\mathscr{A}$;
Return Yes iff $\mathscr{A}$ is synchronizing.

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  Černý in 1964 proved synchronization criterion which allows to find reset word in $O(n^3)$ time.

Given a synchronizing automaton $\mathscr{A}$;

- How to find "relatively" short reset word for $\mathscr{A}$ or its length?
  [Unless $P = NP$], no polynomial time algorithm approximates reset length of $\mathscr{A}$ within a constant factor (CSR 2010).

Exact Decision Variants of The Problem

Check-Eq-Reset-Length | *NP* and *co-NP* hard

Given A synchronizing automaton $\mathscr{A}$ and a positive integer k;
Question: $\mathfrak{C}(\mathscr{A}) = k$?

Unless $\mathrm{NP} = \text{co-NP}$, even non-deterministic polynomial-time algorithms cannot solve the above problem.

Check-Reset-Length | *NP*-complete (Rystsov, Eppstein and others)

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For each ψ of SAT-problem with n variables and m clauses he constructed $Epp(\psi)$ such that
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Examples for two instances

$\Sigma = \{a, b\}, \qquad Q = \{q_{i,j} \mid i \in [1, n+1], j \in [1, m]\} \cup \{z_0\}$

$$q_{i,j}.d = \begin{cases} z_0 & \text{if } (d = a \text{ and } x_j \in c_i) \text{ or } (d = b \text{ and } \overline{x_j} \in c_i), \\ z_0 & \text{if } j = n+1, \\ q_{i,j+1} & \text{otherwise.} \end{cases}$$

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It is clear ψ_1 is satisfiable for the truth assignment $\tau : x_1 = x_2 = 0,\ x_3 = 1$ while ψ_2 is not satisfiable.

The word $v(\tau) = bba$ synchronizes $Epp(\psi_1)$ and the word a^4 is a reset word of minimum length for $Epp(\psi_2)$.

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$$\Sigma = \{a, b\}, \qquad Q = \{q_{i,j} \mid i \in [1, n+1], j \in [1, m]\} \cup \{z_0\}$$

$$q_{i,j}.d = \begin{cases} z_0 & \text{if } (d = a \text{ and } x_j \in c_i) \text{ or } (d = b \text{ and } \overline{x_j} \in c_i), \\ z_0 & \text{if } j = n+1, \\ q_{i,j+1} & \text{otherwise.} \end{cases}$$

$$\psi_1 = (x_3 \vee x_1 \vee x_2) \wedge (\overline{x_1} \vee x_2) \wedge (\overline{x_2} \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3}),$$
$$\psi_2 = (x_1 \vee x_2) \wedge (\overline{x_1} \vee x_2) \wedge (\overline{x_2} \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3}).$$

It is clear ψ_1 is satisfiable for the truth assignment $\tau : x_1 = x_2 = 0,\ x_3 = 1$ while ψ_2 is not satisfiable.

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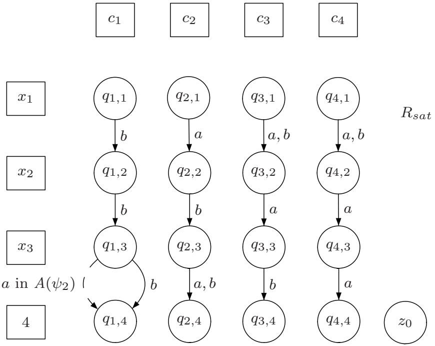
The automata $Epp(\psi_1)$ and $Epp(\psi_2)$

$\psi_1 = ({\color{red}x_3} \vee x_1 \vee x_2) \wedge (\overline{x_1} \vee x_2) \wedge (\overline{x_2} \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3})$

An applied prefix is $v = 1$

(First row)$.v = \{q_{1,1}, q_{2,1}, q_{3,1}, q_{4,1}, z_0\}$



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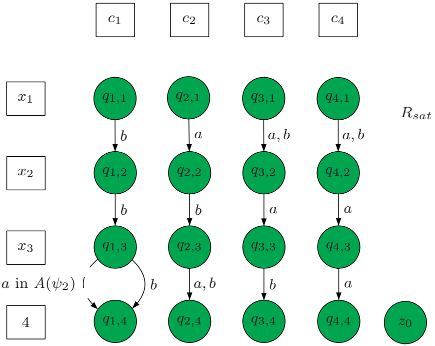

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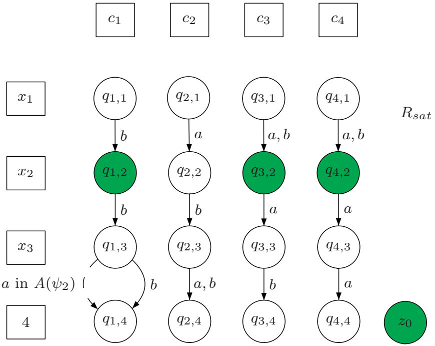

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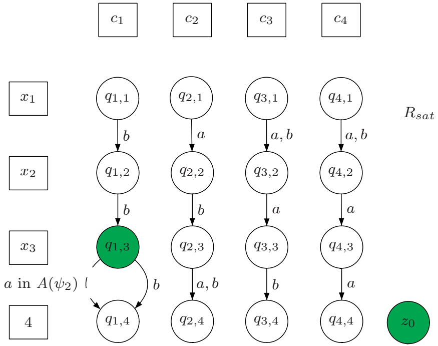

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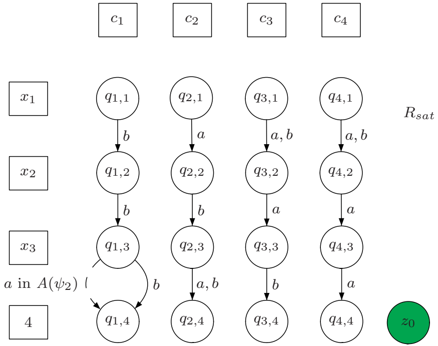

Exact Search Variants of The Problem

Search-Reset-Length | $FP^{NP[log]}$-complete

Given A synchronizing automaton $\mathscr{A}$;
Return $\mathfrak{C}(\mathscr{A})$.

Search-Shortest-Reset-Word | in FP^{NP} and $FP^{NP[log]}$-hard

Given A synchronizing automaton $\mathscr{A}$;
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FP^{NP} and $FP^{NP[log]}$ are complexity classes of search problems that can be solved by a deterministic polynomial time algorithm equipped with an ability to use an oracle for any NP-complete problem by polynomial or logarithmic times respectively.
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Approximation Variant of The Problem

Key Question | Volkov 2008

Can we approximately find the **reset length** within a constant factor in a polynomial time?

An algorithm M **approximates reset length** in $\mathcal{K}$ if for an arbitrary DFA $\mathscr{A} \in \mathcal{K}$, the algorithm calculates a positive integer $M(\mathscr{A})$ such that $M(\mathscr{A}) \geq \mathfrak{C}(\mathscr{A})$.

$\sup\{\frac{M(\mathscr{A})}{\mathfrak{C}(\mathscr{A})} \mid \mathscr{A} \in \mathcal{K}\}$ is an **approximation factor** of M.

Is there a polynomial-time approximation algorithm within a constant factor for Search-Reset-Length?

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The First Result

Theorem 1.

No polynomial-time algorithm approximates the minimum length of reset words within a constant factor.

For every ψ of SAT with n variables we construct synchronizing automaton $\mathscr{A}_r(\psi)$ for $r = 2, 3, \ldots$ such that
$\mathfrak{C}(\mathscr{A}_r(\psi)) \leq n + r$ and $c^{r-1}v(\tau)c$ is reset if ψ is satisfiable on τ,
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The automata $\mathscr{A}(\psi_1)$ and $\mathscr{A}(\psi_2)$

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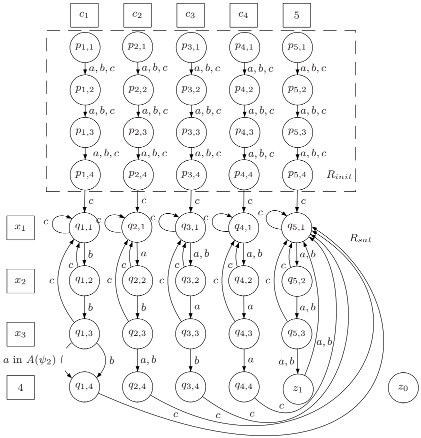

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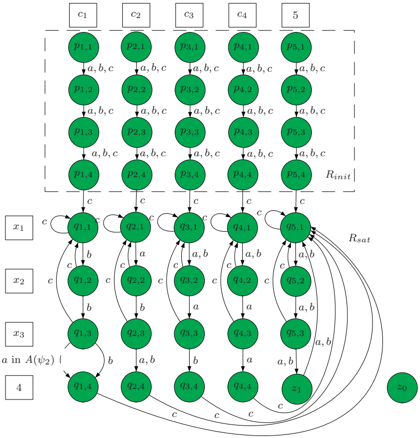

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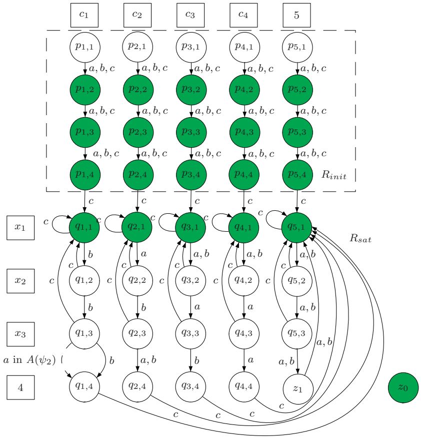

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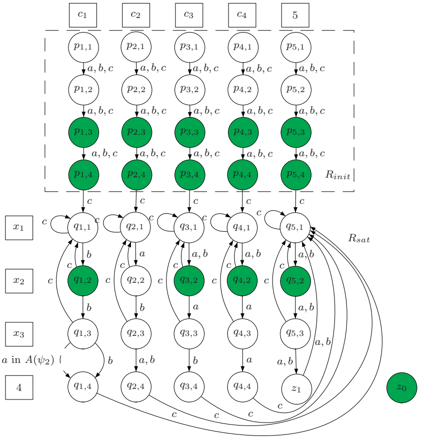

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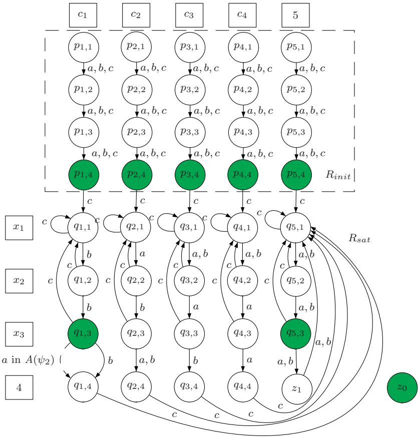

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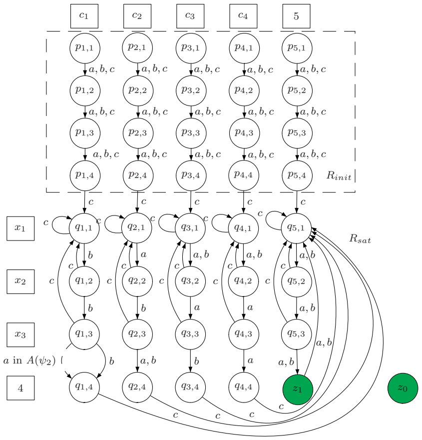

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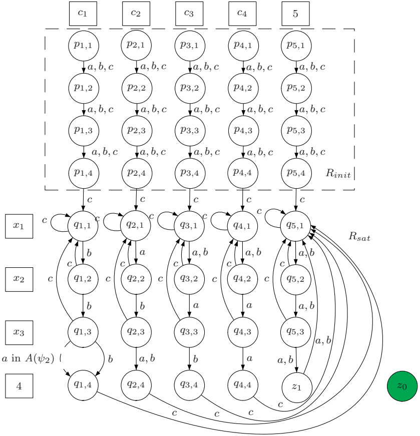
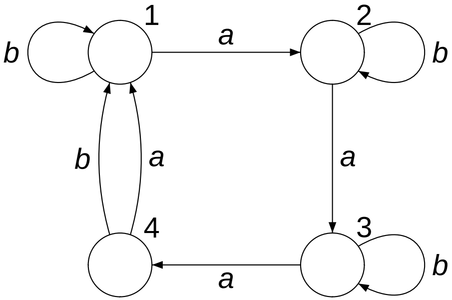
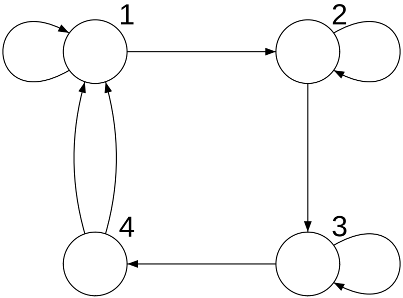




The 4-state Cerny automaton C_4 with shortest reset word ba^3ba^3b of length 9.

Relabeling of Automata and Coloring of Graphs



An underlying graph G_4 of the Cerny automaton C_4.

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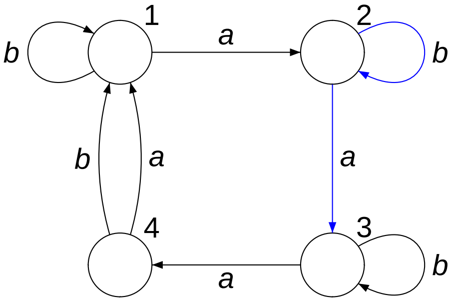

A synchronizing relabeling of C_4 by a permutation of labels on outgoing arrows (from state 2).

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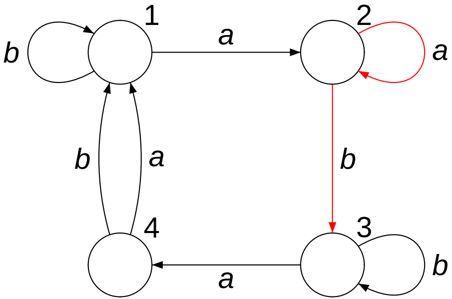

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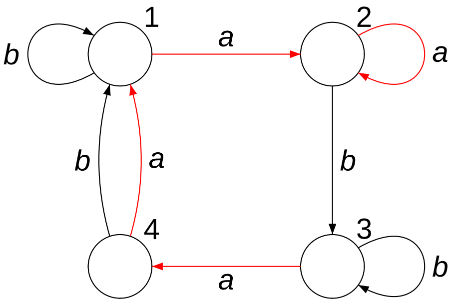

A synchronizing coloring of G_4 with shortest reset word a^3 of length 3.

An example from the real life!

Main Questions And Outline of the Talk

Given an automaton $\mathscr{A}$;

- How to find some reset word for $\mathscr{A}$ if it exists?
  Černý in 1964 proved synchronization criterion which allows to find reset word in $O(n^3)$ time.
- How to relabel $\mathscr{A}$ to make it synchronizing?

Given a synchronizing automaton $\mathscr{A}$;

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Search For Some Synchronizing Coloring

Road Coloring Problem | Adler, Goodwyn, Weiss 1970,77

Does each AGW-graph (strongly connected admissible graph with g.c.d. of cycles length equals one) has a synchronizing coloring?

Particular cases [O'Brien, 1981; Fridman, 1990; Perrin and Schützenberger, 1985; Jonoska N., Suen S., 1995, Carbone A., 2001, J. Kari 2003...]

RCP Solution! | A. Trahtman 2008

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Approximate Optimal Coloring Value

Let $OPT(G)$ denotes the minimal value of $\mathfrak{C}(\mathscr{A}(G))$ for possible colorings $\mathscr{A}(G)$ of AGW-graph G and let us call it optimal coloring value.

A coloring $\mathscr{B}(G)$ with $\mathfrak{C}(\mathscr{B}(G)) = OPT(G)$ is called optimal.

Opt-Coloring-Value

Given An AGW-graph G;
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Key Question | Volkov 2008

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The Second Result

Theorem | Roman 2010

No polynomial-time algorithm exactly finds optimal coloring value.

Theorem 2. | Izvestiya Vuzov (submitted 07.2009)

No polynomial-time algorithm approximates optimal coloring value within a constant factor less than 2.

Can we approximately find the optimal coloring value within a constant factor 2 in a polynomial time?

Proof sketch:
For each ψ of SAT with n variables we construct $G(\psi)$ such that
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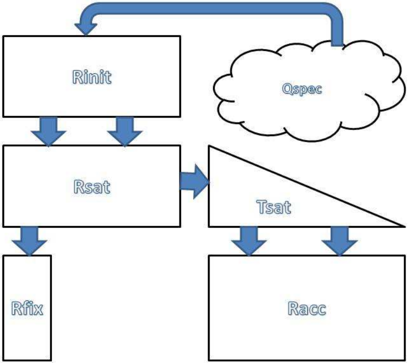
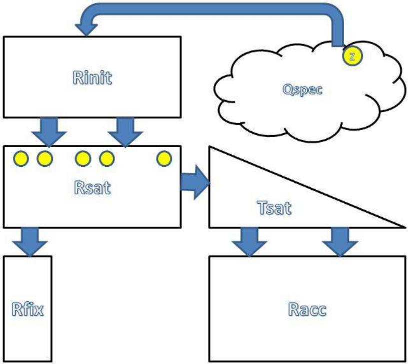
Synchronizing Coloring in GOODCASE



Construction of The Graph $G(\psi)$

Synchronizing Coloring in GOODCASE



$Q.u_{init}$ equals the first row of R_{sat} and state z. The length of u_{init} is $p - p_{small}$.

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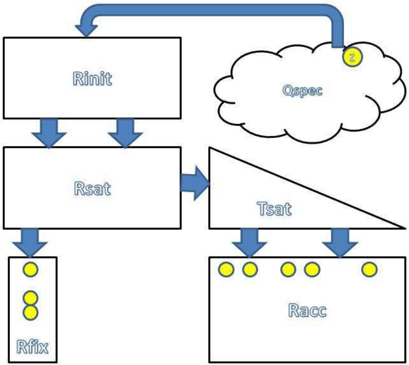

Define $u_{sat} = v(\tau)$. Then $Q.u_{init}u_{sat}$ consists of n states in the first row of R_{acc}, 3 states in R_{fix} with numbers $1, 3, 4$ and state z. The length of u_{sat} is $n + 1$.

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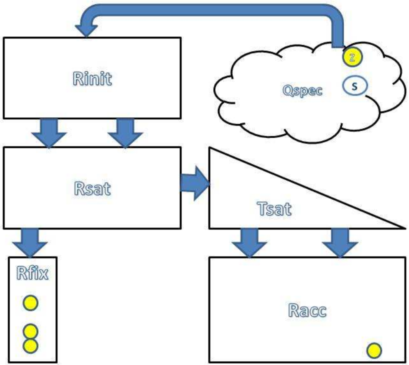
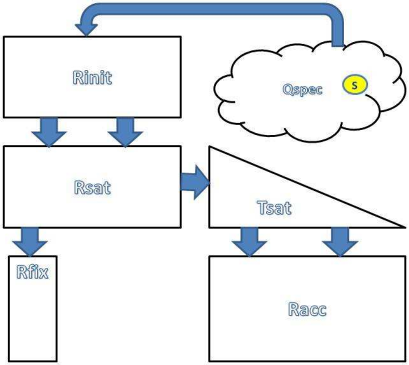


$Q.u_{init}u_{sat}u'_{acc}$ consists of bottom state in R_{acc} and 3 states in R_{fix} with numbers $h-3, h-1, h$ and state z. The length of u'_{acc} is $p_{small}-(n+2)=h$.

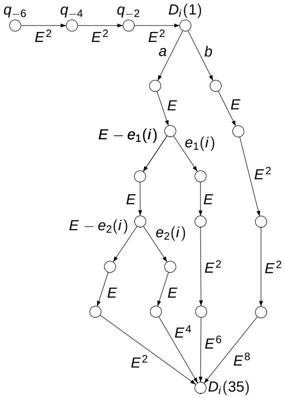
Synchronizing Coloring in GOODCASE



$Q.u_{init}u_{sat}u_{acc} = s$

Counting States in The First Row of R_{acc}

Let $E = \{a, b\}$ and apply a word $\ldots ba^{13} \ldots$ to the green states.



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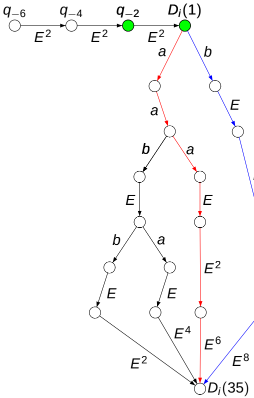

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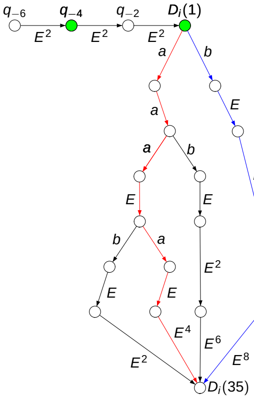

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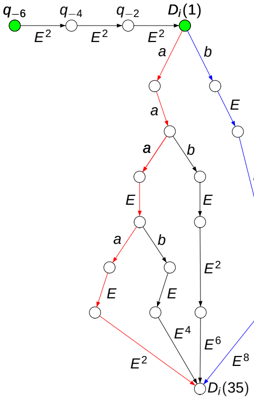
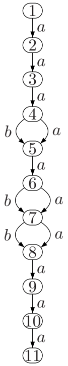


Fixing The Word $w_n(\dots)$ Using Component R_{fix}

Synchronizing Coloring of R_{fix} in GOODCASE.



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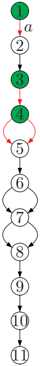
A fixed word $w_n = 1\dots$



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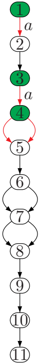

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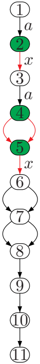

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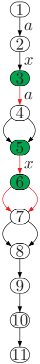

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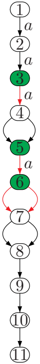

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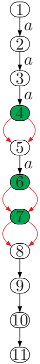

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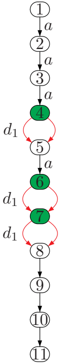

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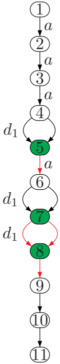

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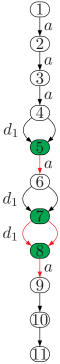

Approximate Optimal Coloring

Opt-Coloring

Given An AGW-graph G;
Return Optimal Coloring of G.

Key Question

Can we approximately find the optimal coloring within a constant factor in a polynomial time?

Remark 1: It doesn't follow from Theorem 2, because we should not find $\mathfrak{C}(\mathscr{A}(G))$ for quasi-optimal coloring $\mathscr{A}(G)$.

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It is sufficient to show how to determine in a polynomial time satisfiability of ψ by coloring of $G(\psi)$ from Theorem 2.

Proof Sketch:

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- Renewal variable values $b_1, b_2, \ldots, b_n$ according to S.

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- Renewal set of states S from the first row of Row_{acc} which can be merged by the word $w_n(d_1, d_2, \ldots, d_n)$.
- It can be done as in the BADCASE using that ψ should be true for the collection $b_1, b_2, \ldots, b_n$.

2-letter alphabet case

- We used a 3-letter alphabets in the Theorems.
- By adding letters with action of letter c, the results extend to any class of automata with bigger alphabet.
- All considered problems are trivial for 1-letter automata.

The results can be extended to the case of 2-letter alphabet.

For each 3-letter automaton $\mathscr{A}$ we can construct a 2-letter automaton $\mathscr{B}$ such that $\mathfrak{C}(\mathscr{A}) \leq \mathfrak{C}(\mathscr{B}) \leq 3\mathfrak{C}(\mathscr{A})$.

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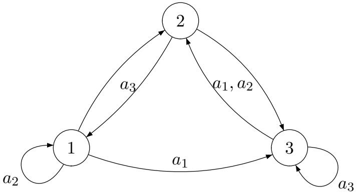
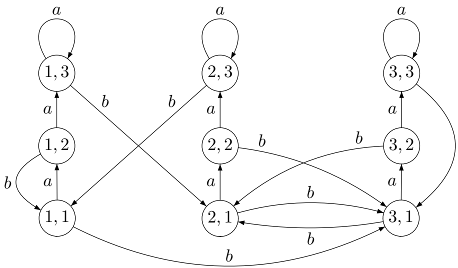
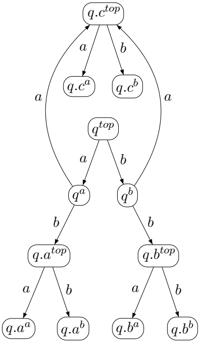
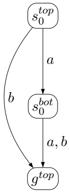
An Automata Transformation in Theorem 1.





Substitutions in Graph $G(\psi)$ in Theorem 2.





- A DFA $\mathscr{A}$ is called an automaton with 0 if it has one immoveable state called 0.
- A **partial finite automaton** (PFA) can have some undefined transitions in difference to DFA.
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Any PFA $\mathscr{B}$ is a result of removing all incoming aroows to 0 for an appropriate DFA $\mathscr{A}$ with 0 and each killing word for $\mathscr{B}$ is reset for $\mathscr{A}$.

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No polynomial-time algorithm can approximate the length of the shortest killing word within a constant factor.

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- The **greedy** algorithm (Eppstein 1990) finds a reset word for n-state automata in $O(n^3)$ time.
- It finds a reset word of length at most $\frac{n^3-n}{6}$ (Pin 1983).
- All experiments with series of slowly synchronized automata generated in our scientific group show it has a logarithmic approximation factor.

Search-LogApprox-Reset-Length(d)

Given An n-state synchronizing automaton $\mathscr{A}$

Return A number between $\mathfrak{C}(\mathscr{A})$ and $d \cdot \log n \cdot \mathfrak{C}(\mathscr{A})$.

Is there a polynomial-time algorithm for the above problem?

Thank you for your attention!

Any questions?