In[1]:=
```
Clear["Global`*"];
ZTransform[b^n*Sin[a*n], n, z]
```

Out[2]=
$$\frac{i\, b \left(-1 + e^{2\, i\, a}\right) z}{2 \left(-b^2\, e^{i\, a} + b \left(1 + e^{2\, i\, a}\right) z - e^{i\, a} z^2\right)}$$

In[3]:=
```
InverseZTransform[3 z / (3 z - 1), z, n]
```

Out[3]=
$$3^{-n}$$

In[4]:=
```
InverseZTransform[b * z / (z - b)^2, z, n]
```

Out[4]=
$$b^n\, n$$

In[5]:=
```
ZTransform[y[n + 1] - 2 * y[n], n, z]
```

Out[5]=
$$-z\, y[0] - 2\, \text{ZTransform}[y[n], n, z] + z\, \text{ZTransform}[y[n], n, z]$$

In[6]:=
```
ZTransform[n, n, z]
```

Out[6]=
$$\frac{z}{(-1 + z)^2}$$

In[7]:=
```
Solve[ZTransform[y[n + 1] - 2 * y[n], n, z] ==
  ZTransform[n, n, z], ZTransform[y[n], n, z]]
```

Out[7]=
$$\left\{\left\{\text{ZTransform}[y[n], n, z] \to \frac{z + z\, y[0] - 2\, z^2\, y[0] + z^3\, y[0]}{(-2 + z)\, (-1 + z)^2}\right\}\right\}$$

In[8]:=
```
InverseZTransform[(z + z y[0] - 2 z^2 y[0] + z^3 y[0])/((-2 + z) (-1 + z)^2), z, n]
```

Out[8]=
$$-1 + 2^n - n + 2^n\, y[0]$$

In[9]:=
```
ZTransform[y[n + 1] - 2 * y[n], n, z]
```

Out[9]=
$$-z\, y[0] - 2\, \text{ZTransform}[y[n], n, z] + z\, \text{ZTransform}[y[n], n, z]$$

In[10]:=
```
ZTransform[3^n, n, z]
```

Out[10]=
$$\frac{z}{-3+z}$$

In[11]:=
```
Solve[ZTransform[y[n + 1] - 2 * y[n], n, z] ==
  ZTransform[3^n, n, z], ZTransform[y[n], n, z]]
```

Out[11]=
$$\left\{\left\{\text{ZTransform}[y[n], n, z] \to \frac{z - 3\,z\,y[0] + z^2\,y[0]}{(-3+z)\,(-2+z)}\right\}\right\}$$

In[12]:=
```
InverseZTransform[(z - 3 z y[0] + z^2 y[0])/((-3 + z) (-2 + z)), z, n]
```

Out[12]=
$$-2^n + 3^n + 2^n\,y[0]$$

In[13]:=
```
ZTransform[y[n + 1], n, z]
```

Out[13]=
$$-z\,y[0] + z\,\text{ZTransform}[y[n], n, z]$$

In[14]:=
```
ZTransform[a * y[n] + b, n, z]
```

Out[14]=
$$\frac{b\,z}{-1+z} + a\,\text{ZTransform}[y[n], n, z]$$

In[15]:=
```
Solve[ZTransform[y[n + 1], n, z] ==
  ZTransform[a * y[n] + b, n, z], ZTransform[y[n], n, z]]
```

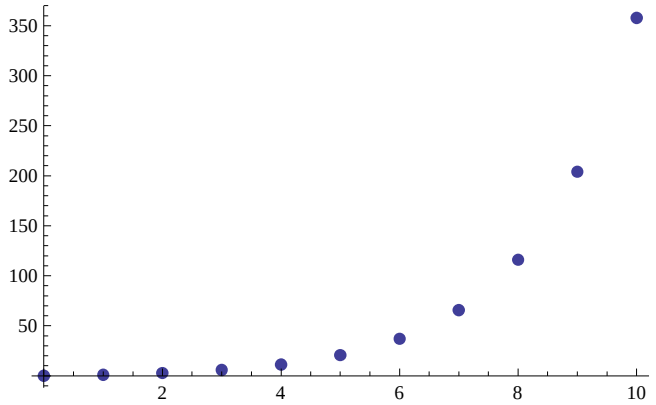
Out[15]=
$$\left\{\left\{\text{ZTransform}[y[n], n, z] \to \frac{b\,z - z\,y[0] + z^2\,y[0]}{(-1+z)\,(-a+z)}\right\}\right\}$$

In[16]:=
```
Simplify[InverseZTransform[(b z - z y[0] + z^2 y[0])/((-1 + z) (-a + z)), z, n]]
```

Out[16]=
$$\frac{(-1+a^n)\,b + (-1+a)\,a^n\,y[0]}{-1+a}$$

In[17]:=
```
a = 7 / 4; b = 1; y[0] = 0; ListPlot[
 Table[{n, ((-1 + a^n) b + (-1 + a) a^n y[0])/(-1 + a)}, {n, 0, 10}],
 PlotStyle → PointSize[0.02]]
```

Out[17]=



In[18]:=
```
LaplaceTransform[Exp[-7 t] * Sin[5 t], t, z]
```

Out[18]=
$$\frac{5}{25 + (7 + z)^2}$$

In[19]:=
```
InverseLaplaceTransform[(24 (1 + 5 z^2 (-2 + z^2)))/(1 + z^2)^5, z, t]
```

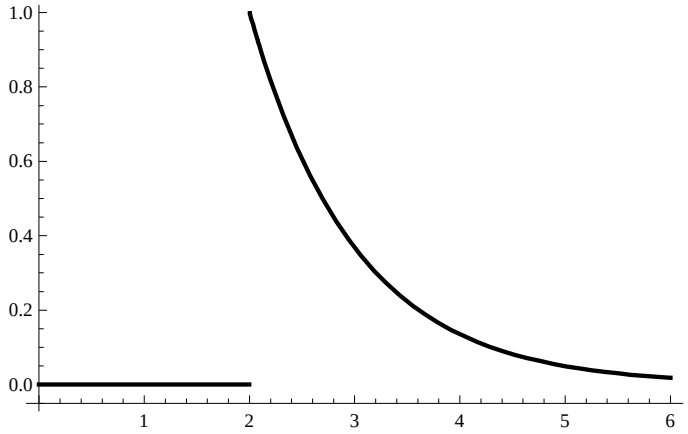
Out[19]=
```
t^4 Sin[t]
```

In[20]:=
```
InverseLaplaceTransform[Exp[-2 z] / (z + 1), z, t]
```

Out[20]=
```
e^(2-t) HeavisideTheta[-2 + t]
```

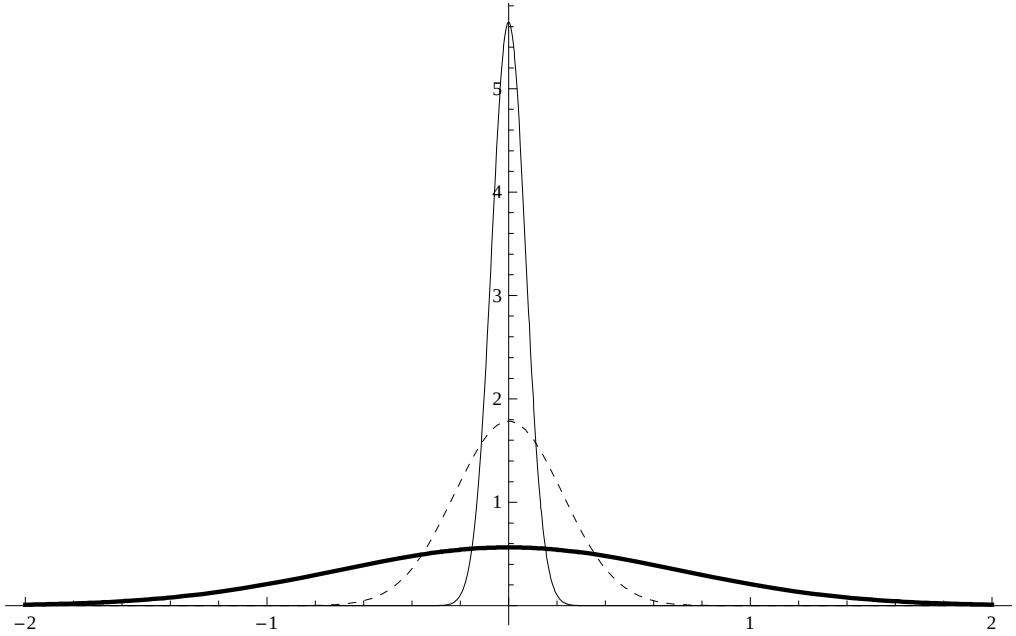
In[21]:=
```
Plot[e^(2-t) UnitStep[-2 + t], {t, 0, 6}, AxesOrigin → {0, -0.05}, PlotStyle → Directive[Black, Thick]]
```

Out[21]=



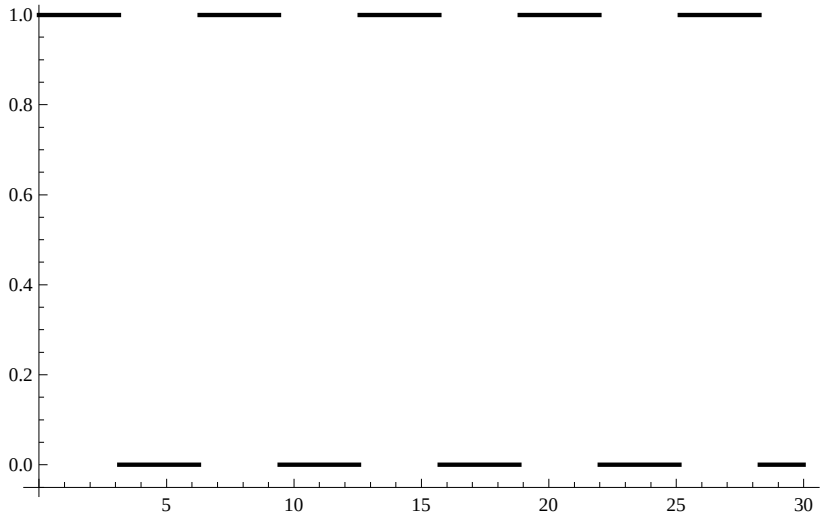
In[22]:=
```
Plot[Evaluate[Sqrt[n / Pi] Exp[-n x^2] /.
    n → {1, 10, 100}], {x, -2, 2}, PlotRange → All, PlotStyle →
  {Directive[Black, Thick], Directive[Black, Dashed], Black}]
```

Out[22]=



In[23]:=
```
Plot[UnitStep[Sin[x]], {x, 0, 30}, AxesOrigin → {0, -0.05}, PlotStyle → Directive[Black, Thick]]
```

Out[23]=



In[24]:=
```
equation = q''[t] + 10 q'[t] + 100 q[t] == DiracDelta[t]
```

Out[24]=
$$100\,q[t] + 10\,q'[t] + q''[t] == \text{DiracDelta}[t]$$

In[25]:=
```
LaplaceTransform[equation, t, z]
```

Out[25]=
$$100\,\text{LaplaceTransform}[q[t], t, z] + z^2\,\text{LaplaceTransform}[q[t], t, z] + 10\,(z\,\text{LaplaceTransform}[q[t], t, z] - q[0]) - z\,q[0] - q'[0] == 1$$

In[26]:=
```
Solve[LaplaceTransform[equation, t, z],
 LaplaceTransform[q[t], t, z]]
```

Out[26]=
$$\left\{\left\{\text{LaplaceTransform}[q[t], t, z] \to \frac{1 + 10\,q[0] + z\,q[0] + q'[0]}{100 + 10\,z + z^2}\right\}\right\}$$

In[27]:=
```
%[[1]] /. {q[0] → 0, q'[0] → 0}
```

Out[27]=
$$\left\{\text{LaplaceTransform}[q[t], t, z] \to \frac{1}{100 + 10\,z + z^2}\right\}$$

In[28]:=
```
InverseLaplaceTransform[LaplaceTransform[q[t], t, z] /. %, z, t]
```

Out[28]=
$$\frac{e^{-5\,t}\,\text{Sin}\left[5\sqrt{3}\;t\right]}{5\sqrt{3}}$$

In[29]:=
```
Plot[e^(-5 t) Sin[5 Sqrt[3] t] / (5 Sqrt[3]), 0.01, {t, 0, 1}, PlotStyle → Directive[Black, Thick]]
```

Out[29]=