```mathematica
In[1]:= Clear["Global`*"];
       Quad[z_, c_] := z^2 + c;

In[3]:= fixeda = z /. Solve[Quad[z, c] == z, z]
```

$$Out[3]= \left\{ \frac{1}{2} \left(1 - \sqrt{1 - 4c} \right), \frac{1}{2} \left(1 + \sqrt{1 - 4c} \right) \right\}$$

```mathematica
In[4]:= Map[2 # &, fixeda]
```

$$Out[4]= \left\{ 1 - \sqrt{1 - 4c}, 1 + \sqrt{1 - 4c} \right\}$$

```mathematica
In[5]:= fixeda = z /. Solve[Nest[Quad[#, c] &, z, 1] == z, z]
```

$$Out[5]= \left\{ \frac{1}{2} \left(1 - \sqrt{1 - 4c} \right), \frac{1}{2} \left(1 + \sqrt{1 - 4c} \right) \right\}$$

```mathematica
In[6]:= fixedb = z /. Solve[Nest[Quad[#, c] &, z, 2] == z, z]
```

$$Out[6]= \left\{ \frac{1}{2} \left(-1 - \sqrt{-3 - 4c} \right), \frac{1}{2} \left(-1 + \sqrt{-3 - 4c} \right), \frac{1}{2} \left(1 - \sqrt{1 - 4c} \right), \frac{1}{2} \left(1 + \sqrt{1 - 4c} \right) \right\}$$

```mathematica
In[7]:= dmap[z_, c_] = D[Nest[Quad[#, c] &, z, 1], z]
```

$$Out[7]= 2z$$

```mathematica
In[8]:= derivs = Expand[Map[dmap[#, c] &, fixeda]]
```
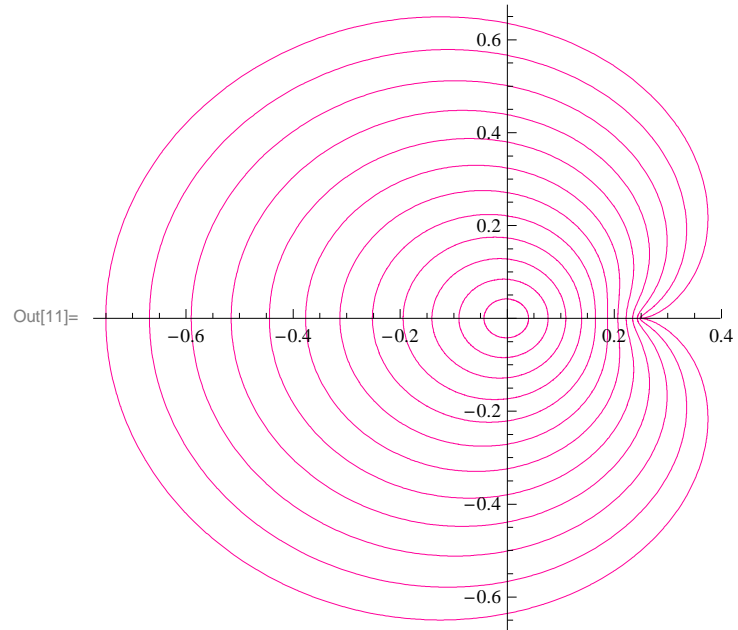
$$Out[8]= \left\{ 1 - \sqrt{1 - 4c}, 1 + \sqrt{1 - 4c} \right\}$$

```mathematica
In[9]:= c /. Solve[derivs[[1]] == μ, c][[1]]
```

$$Out[9]= \frac{1}{4} \left(2\mu - \mu^2 \right)$$

```mathematica
In[10]:= cstable[μ_] := \frac{1}{4} \left( 2\mu - \mu^2 \right);
```

In[11]:= ```
stableregion = ParametricPlot[
    Evaluate[Table[{Re[cstable[r Exp[I θ]]],
      Im[cstable[r Exp[I θ]]]}, {r, 0, 1, 1/12.}]], {θ, 0, 2 Pi},
    AspectRatio -> 1, PlotStyle -> Hue[9/10], PlotRange → All]
```

Out[11]=



In[12]:= ```
fixedb = z /. Solve[Nest[Quad[#, c] &, z, 2] == z, z]
```

Out[12]= $\left\{ \frac{1}{2} \left(-1 - \sqrt{-3 - 4c} \right), \frac{1}{2} \left(-1 + \sqrt{-3 - 4c} \right), \frac{1}{2} \left(1 - \sqrt{1 - 4c} \right), \frac{1}{2} \left(1 + \sqrt{1 - 4c} \right) \right\}$

In[13]:= ```
fixedbonly = Complement[fixedb, fixeda]
```

Out[13]= $\left\{ \frac{1}{2} \left( -1 - \sqrt{-3 - 4c} \right), \frac{1}{2} \left( -1 + \sqrt{-3 - 4c} \right) \right\}$

In[14]:= ```
dmap₂[z_, c_] = D[Nest[Quad[#, c] &, z, 2], z]
```

Out[14]= $4 z \left(c + z^2 \right)$

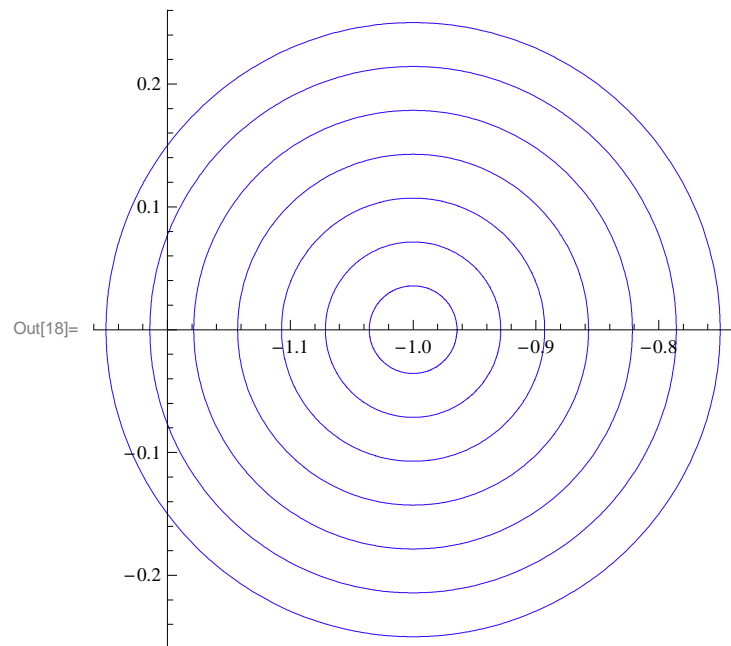In[15]:= **derivsb = Expand[Map[dmap$_2$[#, c] &, fixedbonly]]**

Out[15]= $\{4 + 4\,c,\ 4 + 4\,c\}$

In[16]:= **c /. Solve[derivsb[[1]] == μ, c][[1]]**

Out[16]= $\dfrac{1}{4}\,(-4 + \mu)$

In[17]:= **cstableb[μ_] := (μ - 4) / 4;**

In[18]:= **stableregionb = ParametricPlot[**
   Evaluate[Table[{Re[cstableb[r Exp[I θ]]],
     Im[cstableb[r Exp[I θ]]]}, {r, 0, 1, 1 / 7.}]], {θ, 0, 2 Pi},
   AspectRatio -> 1, PlotStyle -> Hue[7 / 10]]

Out[18]=

```
In[19]:= fixed[n_] := c /. Solve[Nest[Quad[#, c] &, 0, n] == 0, c];
        one = fixed[1];
        none = N[one]
```

```
Out[21]= {0.}
```

```
In[22]:= two = fixed[2]
```

```
Out[22]= {-1, 0}
```

```
In[23]:= twoonly = Complement[two, one];
        ntwo = N[twoonly]
```

```
Out[24]= {-1.}
```

```
In[25]:= three = fixed[3];
        threeonly = Complement[three, one];
        nthree = N[threeonly];
        four = fixed[4];
        fouronly = Complement[four, one, two];
        nfour = N[fouronly];
        five = fixed[5];
        fiveonly = Complement[five, one];
        nfive = N[fiveonly];
        six = fixed[6];
        sixonly = Complement[six, one, two, three];
        nsix = N[sixonly];
        seven = fixed[7];
        sevenonly = Complement[seven, one];
        nseven = N[sevenonly];
        eight = N[fixed[8]];
        eightonly = Complement[eight, N[four], N[two], N[one]];
        neight = N[eightonly];
```

```
In[43]:= data = {none, ntwo, nthree, nfour, nfive, nsix, nseven, neight};
        Map[Length, data]
```

```
Out[44]= {1, 1, 3, 6, 15, 27, 63, 120}
```

```
In[45]:=  aux = Range[1, 8]

Out[45]=  {1, 2, 3, 4, 5, 6, 7, 8}


In[46]:=  plotinfo = Transpose[{aux, data}];
          realplotdata = Map[
             {Hue[1 - #[[1]] / 9], PointSize[0.02], Map[ Point[{Re[#], Im[#]}] &,
                #[[2]]]} &, plotinfo];
```
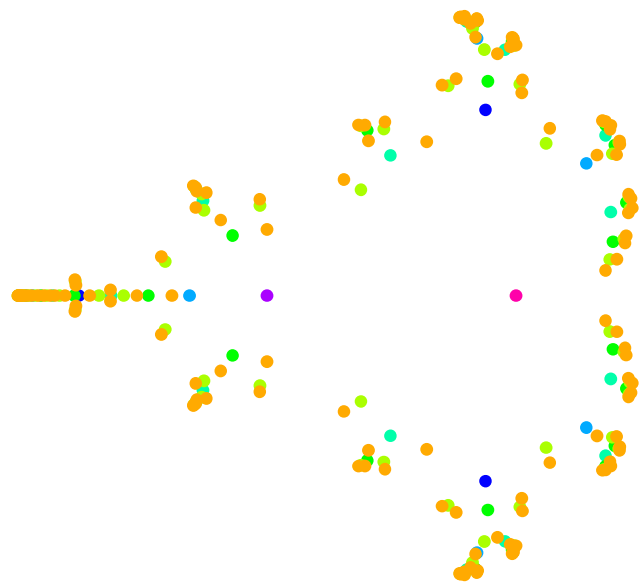
In[48]:= **plota = Show[Graphics[realplotdata]]**

Out[48]=

In[49]:= **fixed[n_, k_] := c /. Solve[Nest[Quad[#, c] &, 0, n] ==**
Nest[Quad[#, c] &, 0, n - k], c];
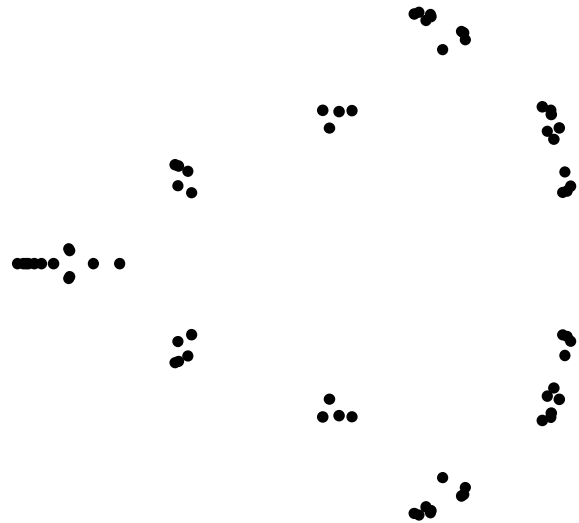
```
In[50]:=  threeone = fixed[3, 1];
          threeoneonly = Complement[threeone, one];
          nthreeoneonly = N[threeoneonly];
          fourone = fixed[4, 1];
          fouroneonly = Complement[fourone, one, threeoneonly];
          nfouroneonly = N[fouroneonly];
          fourtwo = fixed[4, 2];
          fourtwoonly = Complement[fourtwo, one, two, threeoneonly];
          nfourtwoonly = N[fourtwoonly];
          fiveone = fixed[5, 1];
          fiveoneonly = Complement[fiveone, one, threeoneonly, fouroneonly];
          nfiveoneonly = N[fiveoneonly];
          fivetwo = fixed[5, 2];
          fivetwoonly = Complement[fivetwo, two, threeoneonly,
             fouroneonly, fourtwoonly];
          nfivetwoonly = N[fivetwoonly];
          fivethree = fixed[5, 3];
          fivethreeonly = Complement[fivethree, three, threeoneonly, fourtwoonly ];
          nfivethreeonly = N[fivethreeonly];
          sixone = fixed[6, 1];
          sixoneonly = Complement[sixone, one, threeoneonly, fouroneonly,
             fiveoneonly];
          nsixoneonly = N[sixoneonly]; sixtwo = fixed[6, 2];
          sixtwoonly = Complement[sixtwo, two, threeoneonly, fouroneonly, fourtwoonly,
             fiveoneonly, fivetwoonly];
          nsixtwoonly = N[sixtwoonly];
          sixthree = fixed[6, 3];
          sixthreeonly = Complement[sixthree, three, threeoneonly, fouroneonly ,
             fivethreeonly];
          nsixthreeonly = N[sixthreeonly];
          sixfour = fixed[6, 4];
          sixfouronly = Complement[sixfour, one, four(*,five,six*), threeoneonly,
             fourtwoonly];
          nsixfouronly = N[sixfouronly];
```

In[79]:= ```
datab = Flatten[{nthreeoneonly, nfouroneonly,
      nfourtwoonly, nfiveoneonly, nfivetwoonly, nfivethreeonly,
      nsixoneonly, nsixtwoonly, nsixthreeonly, nsixfouronly}];
plotdatab = {PointSize[0.02], Map[
      Point[{Re[#], Im[#]}] &, datab]};
plotb = Show[Graphics[plotdatab],
      PlotRegion -> {{0.05, 0.95}, {0.05, 0.95}}]
```

Out[81]=

In[82]:= **Show[plota, plotb, stableregion, stableregionb,**
  **PlotRegion -> {{0.05, 0.95}, {0.05, 0.95}}, AspectRatio -> 1,**
  **PlotRange -> {{-2, 1 / 2}, {-5 / 4, 5 / 4}}]**

Out[82]=