

Some reflections on (weak) reflections

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The author's reflections will be devoted to three topics: (1) the Orthogonal Subcategory Problem (of P. Freyd and M. Kelly), (2) the Small-Object Argument (of D. Quillen), and (3) life.

Main Results.

Ad (1) Recall the classical theorem of Freyd & Kelly which holds in every “nice enough” category \mathcal{C} : given a class \mathcal{H} of morphisms $\mathcal{H} = \mathcal{S} \cup \mathcal{E}$ where \mathcal{S} is a set and \mathcal{E} consists of epimorphisms, then the orthogonality subcategory of \mathcal{H} is reflective. We* have recently found a surprising example of a locally ranked category which is not “nice enough”.

Ad (2) The above theorem of Freyd & Kelly has a perfect counterpart in case of injectivity. In other words, under the same assumptions we* have proved that the injectivity subcategory of \mathcal{H} is weakly reflective. Moreover, the weak reflection maps are \mathcal{H} -cellular. That is, in “nice enough” categories the Small-Object Argument is valid for all classes $\mathcal{H} = \mathcal{S} \cup \mathcal{E}$ of morphisms.

Ad (3) No main results, work in progress.

*Joint work with Michel Hébert and Lurdes Sousa