Alan S. Cigoli *
Université catholique de Louvain

A relative monotone-light factorization system for internal groupoids

It is a well-known fact that a Barr-exact category $\mathcal{C}$ can be seen as a reflective subcategory of the category $\mathsf{Gpd}(\mathcal{C})$ of its internal groupoids:

$$\mathsf{Gpd}(\mathcal{C}) \underset{D}{\overset{\pi_0}{\underset{\longleftarrow}{\overset{\longrightarrow}{\perp}}}} \mathcal{C} \tag{1}$$

where D sends each object in $\mathcal{C}$ to the corresponding discrete internal groupoid, and π_0 is the connected components functor. This adjunction gives rise to an associated (reflective) factorization system $(\mathcal{E}, \mathcal{M})$, where $\mathcal{E}$ is the class of internal functors inverted by π_0. As we will easily see, this factorization system does not admit an associated *monotone-light* factorization system in the sense of [2].

We will then restrict our attention to the case where $\mathcal{C}$ is also a Mal'tsev category. As explained in [3], in this case the adjunction (1) presents $\mathcal{C}$ as a Birkhoff subcategory of $\mathsf{Gpd}(\mathcal{C})$ and the general theory of central extensions developed in [4] applies here. In particular, central extensions are characterized in [3] as regular epimorphic internal discrete fibrations. We will show that, together with the class of internal final functors, these form a *relative* monotone-light factorization system (in the sense of [1]) for regular epimorphic internal functors.

References:

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