Clemens Berger *
University of Nice (France)

Comprehensive factorisation and non-commutative Stone duality

Lawvere [4] introduced comprehension schemes as a categorical approach to comprehension in set theory. We present them as a possible way of axiomatising the notion of *covering* and obtain a general correspondence between comprehension schemes and factorisation systems. Street and Walters' [3] comprehensive factorisation of a functor between small categories is a special case, but similar factorisation systems exist for topological spaces, groupoids and simplicial sets, cf. [2].

The whole setup is applied to the theory of idempotent semigroups, also known as bands. Surprisingly, many special classes of bands studied in literature get a nice interpretation. For instance, the rectangular bands are precisely the discrete objects and the normal bands are those regular bands whose semilattice reflection is a covering. We introduce *distributive bands* as those bands whose semilattice reflection is an "étale" covering of a distributive lattice, and obtain a contravariant equivalence between the category of right distributive bands and the category of sheaves over spectral spaces. This duality can be considered as a non-commutative version of classical Stone duality between distributive lattices and spectral spaces, resp. Priestley spaces [6]. By restriction we get a duality between Leech's [5] strongly distributive skew lattices and complete spectral sheaves, resp. Priestley sheaves, cf. [1].

References:

[1] A. Bauer, K. Cveto-Vah, M. Gehrke, S. J. van Gool, G. Kudryavtseva – A non-commutative Priestley duality, *Topology Appl.* 160 (2013), 1423–1438.

[2] C. Berger and R. M. Kaufmann – Comprehensive factorisation systems, *Tbilisi Math. J.* 10(3) (2017), 255–277.

[3] R. Street and R. F. C. Walters – The comprehensive factorization of a functor, *Bull. Amer. Math. Soc.* 79 (1973), 936–941.

[4] F. W. Lawvere – Equality in hyperdoctrines and comprehension schema as an adjoint functor, *Proc. Sympos. Pure Math.* 17 (1970), 1–14.

[5] J. Leech – Recent developments in the theory of skew lattices, *Semigroup Forum* 52 (1996), 7–24.

[6] H. A. Priestley – Representation of distributive lattices by means of ordered Stone spaces, *Bull. London Math. Soc.* 2 (1970), 186–190.

*Joint work with Ralph Kaufmann and Mai Gehrke.