

Simon Henry  
Masaryk University, Brno

*A proof of the Simpson conjecture for regular compositions*

In 1991, M. Kapranov and V. Voevodsky published a famously incorrect paper ([4]) which claimed that the homotopy category of spaces (i.e. of “weak  $\infty$ -groupoids”) is equivalent to a homotopy category of strict  $\infty$ -categories in which every arrows is weakly invertible.

In 1998, C. Simpson showed ([5]) that this was false, but conjectured that the claim was correct as soon as one also allows the units (identities) of the  $\infty$ -categories to also be “weak” (but keeping composition, associativity and exchange law strict). He also claimed that Kapranov and Voevodsky’s paper probably contains a proof of this claim and extended further his conjecture to the case of weak  $\infty$ -categories.

In this talk we will explain what is the main obstruction for applying directly Kapranov and Voevodsky’s work to this conjecture, and how it can be lifted. There are some new obstructions that arise when trying to prove the most general version of Simpson’s conjecture from there. But we will present a very recent proof of one particular form of the conjecture: that the homotopy category of spaces is equivalent to a homotopy category of  $\infty$ -groupoids which are defined as  $\omega$ -graphs (globular sets) on which all so-called “regular” compositions are defined, where all the associativity and exchange law of these regular compositions are strictly satisfied, and which have weak units and weak inverses (the more general compositions being defined from these “regular” compositions and the weak units).

This is based on the preprint [1] and two other papers in preparation, which hopefully will be available by the time of the talk ([3], [2]).

REFERENCES:

- [1] Simon Henry. Non-unital polygraphs form a presheaf category. *arXiv preprint arXiv:1711.00744*, 2017.
- [2] Simon Henry. Regular polygraphs and the Simpson conjecture. *to appear*, 2018.
- [3] Simon Henry. Weal model categories. *to appear*, 2018.
- [4] Mikhail M Kapranov and Vladimir A. Voevodsky.  $\infty$ -groupoids and homotopy types. *Cahiers de Topologie et Géométrie Différentielle Catégoriques*, 32(1):29–46, 1991.
- [5] Carlos Simpson. Homotopy types of strict 3-groupoids. *arXiv preprint math/9810059*, 1998.