Matrices & Operators
Workshop with Abraham Berman

BOOK OF ABSTRACTS

Department of Mathematics,
University of Coimbra, Portugal
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Part I

Contributed Talks

On the inverse field of values problem

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Abstract

The field of values of a linear operator is the convex set in the complex plane comprising all Rayleigh quotients. For a given complex matrix, Uhlig proposed the inverse field of values problem: given a point inside the field of values determine a unit vector for which this point is the corresponding Rayleigh quotient. In the present note we propose an alternative method of solution to those that have appeared in the literature. Our approach builds on the fact that the field of values can be seen as a union of ellipses under a compression to the bidimensional case, in which case the problem has an exact solution.

Keywords: Field of values, inverse problem, generating vector, compression

Co(mpletely) positive matrices and optimization

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Abstract

We describe known results and open questions on the dual cones of completely positive matrices and copositive matrices and on their applications in optimization.

One sided invertibility of matrices and Toeplitz operators

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Abstract

Conditions under which Fredholmness, Coburn's property and invertibility are shared by a Toeplitz operator with matrix symbol G and the Toeplitz operator with scalar symbol $\det G$ are presented. These results are based on one-sided invertibility criteria for rectangular matrices over appropriate commutative rings and related scalar corona type problems.

The talk is based on joint work with L. Rodman and I. Spitkovsky.

Star sets, star complements and graphs with convex-qp stability number

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Abstract

Consider a graph G with n vertices and an adjacency eigenvalue λ (simply called an eigenvalue of G). Let P be the matrix of the orthogonal projection of $\mathbb{R}^n$ onto the eigenspace of λ, $\mathcal{E}_G(\lambda)$, with respect to the standard orthonormal basis $\{e_1, \dots, e_n\}$ of $\mathbb{R}^n$. Then the set of vectors Pe_j $(j = 1, \dots n)$ spans $\mathcal{E}_G(\lambda)$ and therefore there exists $X \subseteq V(G)$ such that the vectors Pe_j $(j \in X)$ form a basis for $\mathcal{E}_G(\lambda)$. Such a set X is called a *star set* for λ in G or simply a λ-star set of G and $\bar{X} = V(G) \setminus X$ is said a λ-*co-star set* of G, while $G - X = G\left[\bar{X}\right]$ is called a *star complement* for λ in G. If G has m distinct eigenvalues $\mu_1 \geq \cdots \geq \mu_m$, where each eigenvalue μ_i has multiplicity $k_i, i = 1, \dots, m$ (and then $\sum_{i=1}^{m} k_i = n$), it can be proved that there is a partition $X_1 \cup \cdots \cup X_m$ of $V(G)$ where each part X_i is a μ_i-star set (and then has cardinality k_i). This partition is called a *star partition* of G.

The graphs for which the stability number, that is, the size of a stable set (a set of mutually non-adjacent vertices) of maximum cardinality, can be determined solving a convex quadratic program are called *graphs with convex-qp stability number*, where *qp* means quadratic programming. The graphs with convex-*qp* stability number are called $\mathcal{Q}$-graphs.

In this presentation, several combinatorial properties of $\mathcal{Q}$-graphs, G, are highlighted and a few relations between star complements of the least eigenvalue of G and its maximum stable sets are presented. As a consequence, a simplex-like approach to the recognition of $\mathcal{Q}$-graphs is described.

A joint work with Carlos J. Luz.

Keywords: Graph spectra, star sets and star complements, stability number, graphs with convex-*qp* stability number.

Higher order derivatives and norms of certain matrix functions

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Abstract

There is a formula for the first order derivative of the determinant known as the *Jacobi formula*,

$$D\det(A)(X) = \mathrm{tr}(\mathrm{adj}(A)^T X).$$

Recently, T. Jain and R. Bhatia have obtained formulas for the higher order derivatives of the determinant and for the m-th compound of the $n\times n$ matrix $A : \wedge^m A$, later P. Grover also calculated formulas for the permanent and for the induced power of a matrix $\vee^m A$. It is known that the determinant and the permanent are particular cases of the immanant map. On the other hand $\wedge^m A$ and $\vee^m A$ are special cases of the χ-symmetric tensor power of A. In this talk we present formulas for the higher order derivatives and for the norms of all immanants and all symmetric tensor powers.

Sets of Parter Vertices which are Parter Sets

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Abstract

Given an Hermitian matrix, whose graph is a tree, having a multiple eigenvalue λ, the Parter-Wiener theorem guarantees the existence of principal submatrices for which the multiplicity of λ increases. The vertices of the tree whose removal give rise to these principal submatrices are called weak Parter vertices and with some additional conditions are called Parter vertices. A set of k Parter vertices whose removal increase the multiplicity of λ by k is called Parter set. As observed by several authors a set of Parter vertices is not necessarily a Parter set. We prove that if A is a symmetric matrix, whose graph is a tree, and λ is an eigenvalue of A whose multiplicity does not exceed 3, then every set of Parter vertices, for λ relative to A, is also a Parter set.

Keywords: Parter vertices, Parter set, eigenvalues, tree.

AMS Subject of Classification: 15A18, 15A57, 05C50

References

1. Fernandes, R., Cruz, H. F. da (2014). Sets of Parter Vertices which are Parter Sets, *Linear Algebra and its Applications* **448**, pp. 37–54.
2. Johnson, C. R., Leal Duarte, A., Saiago, C. M., Sutton, B. D., Witt, A. J. (2003). On the relative position of multiple eigenvalues in the spectrum of an Hermitian matrix with a given graph, *Linear Algebra and its Applications* **363**, pp. 147–159.
3. Johnson, C. R., Leal Duarte, A., Saiago, C. M. (2003). The Parter-Wiener theorem: refinement and generalization, *Siam J. Matrix Anal. Appl.* **25**, pp. 352–361.
4. Johnson, C. R., Sutton B. D. (2004). Hermitian matrices, eigenvalue multiplicities and eigenvector components, *Siam J. Matrix Anal. Appl.* **26**, pp. 390–399.
5. Parter, S. (1960). On the eigenvalues and eigenvectors of a class matrices, *J. Soc. Indust. Appl. Math.* **8**, pp. 376–388.
6. Wiener, G. (1984). Spectral multiplicity and splitting results for a class of qualitative matrices, *Linear Algebra Appl.* **61**, pp. 15–29.

Zero in the closure of the numerical range

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Abstract

Let $\mathscr{W}_{\{0\}}$ be the set of all operators which contain 0 in the closure of the numerical range, that is,

$$\mathscr{W}_{\{0\}} = \{A \in B(H);\ 0 \in \overline{W(A)}\}.$$

Some properties of this set of operators will be presented and we will discuss a less obvious algebraic structure of $\mathscr{W}_{\{0\}}$. Namely, for some particular sets $\mathcal{T} \subseteq B(H)$, we are able to characterize the set of operators $A \in B(H)$ such that $TA \in \mathscr{W}_{\{0\}}$, for every $T \in \mathcal{T}$.

Keywords: Numerical Range.

References

1. Bračič, J., Diogo, C., *Operators with a given part of the numerical range*, Math. Slovaca, to appear.

Upper bounds on the magnitude of solutions of certain linear systems

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Abstract

We consider a linear homogeneous system of m equations in n unknowns with integer coefficients over the reals. Assume that the sum of the absolute values of the coefficients of each equation does not exceed $k + 1$ for some positive integer k. We show that if the system has a nontrivial solution then there exists a nontrivial solution $x = (x_1, \ldots, x_n)$ such that $|x_j|/|x_i| \leq k^{n-1}$ for each i, j satisfying $x_i x_j \neq 0$. This inequality is sharp. We also prove a conjecture of A. Tyszka related to our results.

This is a joint work with S. Friedland and G. Porta.

Block-symmetric Fiedler pencils with repetition and strong linearizations of matrix polynomials

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Abstract

Let $P(\lambda)$ be a matrix polynomial of degree k, whose coefficients are n-by-n matrices with entries in a field F. S. Vologiannidis and E. N. Antoniou (2011) introduced the family of Fiedler pencils with repetition associated with $P(\lambda)$.

In this talk we describe the Fiedler pencils with repetition that are block-symmetric. In particular, these pencils are symmetric when $P(\lambda)$ is. We give conditions under which they are strong linearizations of $P(\lambda)$. These linearizations are companion forms in the sense that their coefficients can be viewed as k-by-k block matrices and each n-by-n block is either 0, $\pm I_n$, or $\pm A_i$, where A_i, $i = 0, \ldots, k$, are the coefficients of $P(\lambda)$.

Keywords: Block-symmetric Fiedler pencils with repetition, companion form, matrix polynomial, polynomial eigenvalue problem, Symmetric linearization.

Independent sets for irreducible characters of S_n

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Abstract Let λ be an irreducible symmetric character of S_n. Let $g \to A(g)$ be a matricial representation affording λ. Let T be the set of the transpositions of S_n and $A(T) = \{A(g) : g \in T\}$. Let

$$\lambda = \left(m^{m-t}, (m+k-t)^k, (m-t)^{t-k}\right), \, m > t \geq k \geq 0.$$

In this paper we prove that the set $A(T)$ is linearly dependent.

Infimum of two projections

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Abstract If E and F are non-commutative projections with ranges M and N, then it is known that common algebraic operations are not sufficient to find the projection with range $M \cap N$, expressed in terms of E and F.

In this work a solution to this problem is presented. Such solution is given as an application of generalized inverse theory, considering the extension to the singular case of the definition of parallel sum of matrices .

Keywords: Projections, parallel sums, generalized inverses.

References

1. Anderson, W. N., Duffin, R. J. (1969). Series and parallel addition of matrices, *Journal of Math. Anal. Appl.* **26**, pp. 576–594.
2. Filmore, P. A., Williams, J. P. (1971). On operator ranges, *Adv. in Math.* **7**, pp. 254–281.
3. Halmos, P. R. (1967). *A Hilbert Space Problem Book.* New York, Heidelberg, Berlin: Springer-Verlag.

The 123 Theorem of Probability Theory and Copositive Matrices

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Abstract

Alon and Yuster give for independent identically distributed real or vector valued random variables X, Y combinatorially proved estimates of the form $\text{Prob}(\|X-Y\| \leq b) \leq c\,\text{Prob}(\|X-Y\| \leq a)$. We derive these inequalities in the cases that they assume only finitely many values using copositive matrices instead. Classical criteria like those given by Cottle, Habetler and Lemke and Martin are invoked. The extension to arbitrary random variables is done using measure theoretic considerations; more precisely a variety of facts found in the books by Bauer and Loève are used. We also formulate a version of this inequality as an integral inequality for monotone functions. After submission of a paper with above results we found that a paper by Siegmund-Schultze and von Weizsaecker proves the inequality $P(|X+Y| \leq 1) < 2P(|X-Y| \leq 1)$ as one of its main tools. As far as we see in the moment, it allows a similar approach using copositive matrices.

Keywords: Copositive matrices, independent identical distribution, randomvariables.

References

1. Alon, N., Yuster, R. (1995). The 123 Theorem and Its Extensions, *J. of Combin. Theory Ser. A*, **72**, pp. 321–331.
2. Bauer, H. (1981). *Probability theory and elements of measure theory*, Academic Press.
3. Cottle, R. W., Habetler C. E., Lemke, G. J. (1970). On classes of copositive matrices, *Linear Algebra Appl.*, **3**, pp. 295–310.
4. Loève, M. (1977). *Probability Theory I*, 4th Edition, Springer.
5. Martin, D. H. (1981). Finite criteria for conditional positivity of a quadratic form, *Linear Algebra Appl.*, **39**, pp. 9–21.
6. Siegmund-Schultze, R., Weizsäcker, H. von (2007). Level crossing probabilities. I: One-dimensional random walks and symmetrization. *Adv. Math.*, **208**, pp. 672–679.

On Milne and Ostrowski Inequalities of Aczél type

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Abstract

Milne's inequality is a refinement of Cauchy-Schwarz inequality. Another classical inequality for sequences of real numbers is due to A. M. Ostrowski. In a Lorentz space, the Schwarz inequality becomes reversed and, for real timelike vectors, it is known as Aczél inequality. In this talk, a operator inequality of Aczél type for operator means is presented and a Milne's type interpolation of Aczél inequality is easily derived. An indefinite version of the Ostrowski inequality is obtained, as well as its generalization for J-Gramians and some further related inequalities.

Keywords: Milne inequality, Ostrowski inequality, Aczél inequality, Lorentz space, reverse Schwarz inequality, timelike vectors.

References

1. Bebiano, N., Lemos, R. and Providência, J. da (2012). On a reverse Heinz-Kato-Furuta inequality *Linear Algebra Appl.* **437**, pp. 1892–1905.
2. Murnaghan, F. D. (1950). Schwarz' inequality and Lorentz spaces, *Proc. N. A. S.* **36**, pp. 673–676.
3. Ostrowski, A. M. (1951). *Vorlesungenüber Ãijber Differential und Integralrechnung II.* Basel: Birkhauser.

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Lexicographical combinatorial generation and Gray codes for noncrossing and nonnesting set partitions of types A and B.

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Abstract

A Gray code is a listing structure for a set of combinatorial objects such that some consistent (usually minimal) change property is maintained throughout adjacent elements in the list. I shall present combinatorial Gray codes and explicit designs of efficient algorithms for lexicographical combinatorial generation of the sets of noncrossing and nonnesting set partitions of length n and types A and B.

This is a joint work with Alessandro Conflitti (CMUC).

An extension of a Fiedler's lemma and its application on graph energy

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Abstract

In this talk it is presented an extension of a Fiedler's lemma introduced in [1], and this extension is applied to the determination of eigenvalues of graphs belonging to a particular family and also to the determination of the graph energy (including lower and upper bounds), [2]. Some additional consequences applied to generalized composition of graphs are presented, [3].

Keywords: Graph spectra, graph energy.

References

1. Fiedler, M. (1974). Eigenvalues of nonnegative symmetric matrices, *Linear Algebra Appl.* **9**, pp. 119–142..
2. Cardoso, D. M., Gutman, I., Martins, E. A., Robbiano, M. (2011). A Generalization of Fiedler's lemma and some applications, *Linear and Multilinear Algebra* **59 (8)**, pp. 929–942.
3. Cardoso, D. M., Freitas, M. A. A. de, Martins, E. A., Robbiano, M. (2013). Spectra of graphs obtained by a generalization of the join graph operation, *Discrete Mathematics*, **313 (5)**, pp. 733–741

2 by 2 matrices and formal degree for L-parameters of SL_2 over a local function field

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Abstract

Let K be a local function field. Following André Weil's famous paper Exércices dyadiques we study a parametrization of L-parameters for representations of SL_2 over K and compute the associated formal degrees.

This is a joint work with Roger Plymen.

Coupled cell networks: mixing digraphs, matrices and dynamics

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Abstract

In the theory of coupled cell networks, formalized by Ian Stewart, Martin Golubitsky and coworkers, a cell is a dynamical system and a coupled cell system is a finite collection of interacting cells. A coupled cell system can be associated with a network - a directed graph whose nodes represent cells and whose arrows represent couplings between cells. The dynamical connectivity between the distinct cells of a regular network is represented by an adjacency matrix.

In this talk we present some important applications of the theory of digraphs and of matrices to the theory of coupled cell networks. We pay special attention to the cellular splitting process and to bifurcations occurring in coupled cell systems.

Keywords: Coupled cell networks, digraphs, matrices, dynamics, bifurcations.

Numerical ranges and compressions

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Abstract A characterization of the numerical range as a union of ellipses under a compression to the two-dimensional case, in which case the problem has an exact solution, is obtained. Refining an idea of Marcus and Pesce [1], we provide two alternative algorithms to plot the numerical range of a general complex matrix, which perform faster and more accurately than the existing ones. Two Matlab implementation of the algorithms are included.

Keywords: Numerical range, compression.

References

1. Marcus, M., Pesce, C. (1987). Computer generated numerical ranges and some resulting theorems, *Linear and Multilinear Algebra*, **20** pp. 121–157.
2. Maroulas, J., Adam, M. (1999). Compressions and dilations of numerical ranges, *SIAM J. Matrix Anal. Appl.*, **21, No. 1**, pp. 230–244.

The group inverse of a 2×2 block matrix

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Abstract

Recent results have characterized the existence of the group inverse of 2×2 block matrices with a zero (2,2) block in terms of its blocks. This problem is called the 220 group inverse problem. These results have been very recently extended to the case the (2,2) block has a group inverse, not being necessarily zero.

In this talk, we will consider 2×2 block matrices over a general (not necessarily von Neumann regular) ring, assuming some local regularity on the elements. We will use outer inverses and the Brown-McCoy shift to characterize the existence of the inverse and group inverse of such block matrices.

Keywords: Group inverse, regular ring, generalized inverse, Outer inverses, reflexive inverses, block matrix.

References

1. Brown, Bailey, McCoy, Neal H. (1950). The maximal regular ideal of a ring. *Proc. Amer. Math. Soc.* **1**, pp. 165–171.
2. Castro-González, N., Robles, J., Vélez-Cerrada, J.Y. (2013). The group inverse of 2×2 matrices over a ring. *Linear Algebra and Appl.* **438, no. 9**, pp. 3600–3609.
3. Hartwig, R.E.; Patrício, P.; Some Regular Sums, to appear in *Linear and Multilinear Algebra*.
4. Hartwig, R.E.; Patrício, P.; Two by two units, *submitted*.
5. Patrício, P.; Hartwig, R.E. (2010). The (2,2,0) group inverse problem. *Appl. Math. Comput.* **217, no. 2**, pp. 516–520.
6. Puystjens, R., Hartwig, R.E. (1997). The group inverse of a companion matrix. *Linear and Multilinear Algebra* **43 , no. 1-3**, pp. 137–150.

Orthogonal polynomials, tridiagonal matrices, and spectral theory of Jacobi operatotrs

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Abstract

In this talk we present some results on the spectral theory of tridiagonal matrices and Jacobi operators, focusing specially on the so-called tridiagonal $k-$Toeplitz matrices and some perturbations of them. These results are obtained by using well known connections between tridiagonal matrices and orthogonal polynomial sequences (OPS), applying a general theory of OPS related to polynomial mappings. For Jacobi operators, we apply the analytic theory of OPS to state a relation between the positive Borel measures with respect to which the involved OPS are orthogonal. Indeed, the mentioned polynomial mapping allows us to state the spectral properties of the involved matrices, by identifying the spectra (or the essential spectra) of the associated operators with the supports of the orthogonality measures. Some examples will be presented.

Keywords: Orthogonal polynomials, polynomial mappings, tridiagonal matrices, Jacobi operators.

References

1. Álvarez-Nodarse, R., Petronilho, J., Quintero, N. R. (2012). Spectral properties of certain tridiagonal matrices. *Linear Algebra Appl.* **436**, pp. 682–698.
2. de Jesus, M.N., Petronilho, J. (2011). Spectra of certain Jacobi operators. *J. Phys. A: Math. Theor.* **44**, 375203 (20pp).
3. de Jesus, M.N., Petronilho, J. (2010). On orthogonal polynomials obtained via polynomial mappings. *J. Approx. Theory* **162**, pp. 2243–2277.
4. Álvarez-Nodarse, R., Petronilho, J., Quintero, N. R. (2005). On some tridiagonal $k-$Toeplitz matrices: algebraic and analytical aspects. Applications. *J. Comput. Appl. Math.* **184**, pp. 518–537.
5. da Fonseca, C.M., Petronilho, J. (2005). Explicit inverse of a tridiagonal $k-$Toeplitz matrix. *Numer. Math.* **100**, pp. 457–482.
6. Marcellán, F., Petronilho, J. (1997). Eigenproblems for tridiagonal $2-$Toeplitz matrices and quadratic polynomial mappings. *Linear Algebra Appl.* **260**, pp. 169–208.

Part II

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Part III

Appendix

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