Discrete order statistics and their applications in reliability theory

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Order statistics play an important role in reliability theory. They appear in a natural way when we consider lifetimes of some technical structures. Let $X_1, X_2, \dots, X_n$ be lifetimes of n items that form a system and $X_{1:n} \le X_{2:n} \le \dots \le X_{n:n}$ be the corresponding order statistics. Then it is easily seen that

- $X_{1:n}$ describes the lifetime of a series system,
- $X_{n:n}$ describes the lifetime of a parallel system,
- $X_{k:n}$ describes the lifetime of a k-out-of-n system, i.e. a system consisting of n components that functions if and only if at least k of the components work.

More generally, the lifetime of any coherent system can be expressed via order statistics, provided the random vector $(X_1, X_2, \dots, X_n)$ is exchangeable, i.e. $(X_1, X_2, \dots, X_n)$ has the same distribution as $(X_1, X_2, \dots, X_n)$ for any permutation $(\sigma(1), \sigma(2), \dots, \sigma(n))$ of $(1, 2, \dots, n)$. A coherent system is a technical structure such that all its components are relevant and having the property that replacing a failed component by a working one cannot cause a working system to fail. As shown in [10], for any coherent system with exchangeable lifetimes of components, the lifetime of the system T is given by

$$P(T > t) = \sum_{i=1}^{n} s_i P(X_{i:n} > t) \text{ for any } t,$$

where $\mathbf{s} = (s_1, \dots, s_n)$ is the Samaniego signature of the system.

In my talk, I will focus on the case when the lifetimes $X_1, X_2, \dots, X_n$ are discrete random variables. As an example of a system with discrete X_i, $i = 1, 2, \dots, n$, we can consider a system in which the component lifetimes are the numbers of turn-on and switch-off up to the failure, or a system which performs a task repetitively and its components have certain probabilities of failure upon each cycle. Treating the discrete case is considerably more complicated than the absolutely continuous one due to non-zero probability of ties among times of component failures.

I will present methods for studying reliability properties of k-out-of-n systems composed of components with not necessarily independent and not necessarily identically distributed discrete lifetimes. I will also explain how some of these methods can be generalized to the class of coherent systems with exchangeable components.

First, I will show how to obtain a closed-form formula describing the joint probability mass function of any subset of order statistics arising from discrete samples. This formula will be a starting point to examine the following problems of interest:

- probability of a failure of a system at a given time;
- various residual lifetimes of a used system;
- residual lifetimes of components that survived system failure;
- moments of lifetime of a system;
- the distribution of the number of failed components at the time of a failure of the system;
- inference based on failure times of components observed up to and including system failure.

Discussing the above mentioned problems I will show various probabilistic properties of discrete order statistics, including characterizations and asymptotic behavior. I will also point out some open problems.

My talk will be based on a series of papers [1]-[9].

References

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