An anti-classification theorem for the topological conjugacy problem for Cantor minimal systems

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The isomorphism problem in dynamics dates back to a question of von Neumann from 1932: Is it possible to classify the ergodic measure-preserving diffeomorphisms of a compact manifold up to isomorphism? Foreman, Rudolph and Weiss proved an *anti-classification theorem* that rigorously explains why, in a certain sense, such a classification is impossible [1]. We study a topological analogue of von Neumann's problem. Let $\mathrm{Min}(C)$ stand for the Polish space of all minimal homeomorphisms of the Cantor set C. Recall that a homeomorphism $T\colon C \to C$ is minimal if every orbit of T is dense in C. We say that S and T in $\mathrm{Min}(C)$ are topologically conjugate if there is a homeomorphism $h\colon C \to C$ such that $S \circ h = h \circ T$. We will discuss an anti-classification result, saying that it is impossible to tell if two minimal Cantor set homeomorphisms are topologically conjugate using only a countable amount of information and computation. We will explain, how to understand such an anti-classification result and why it suffices to show that the topological conjugacy relation of Cantor minimal systems treated as a subset of $\mathrm{Min}(C) \times \mathrm{Min}(C)$ is complete analytic, so a non-Borel subset of $\mathrm{Min}(C) \times \mathrm{Min}(C)$.

References

[1] Matthew Foreman, Daniel J. Rudolph, Benjamin Weiss, *The conjugacy problem in ergodic theory.* Ann. of Math. (2) **173** (2011), no. 3, 1529–1586.

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