Quantale-enriched lower separation axioms and the principle of enriched continuous extension

Igor Arrieta*

School of Computer Science
University of Birmingham, UK
i.arrietatorres@bham.ac.uk

Quantale-enriched topological spaces provide a convenient framework to unify constructions from different areas of mathematics such as closed left ideal lattices of non-commutative C^*-algebras, approach spaces, or quantale-valued topological spaces (see [2]).

In this talk, we present a quantale-enriched version of the lower separation axioms, with the goal of proving the principle of enriched continuous extension for quantale-enriched topological spaces. This includes a version of the regularity axiom based on the cocompleteness of $\mathfrak{Q}$-enriched topologies viewed as $\mathfrak{Q}^t$-enriched categories and requires the concept of closed $\mathfrak{Q}$-enriched presheaves. Moreover, forced by the non-idempotency of the quantale multiplication we also present a completely new and weaker form of regularity, under which the Kolmogoroff, Fréchet and Hausdorff separation axioms are all equivalent. Further, and more importantly, weak regularity is sufficient for the formulation of the principle of continuous extension.

As a remarkable result, among other things, we point out that in the case of a commutative Girard quantale $\mathfrak{Q}$, every projective $\mathfrak{Q}$-module in Sup provided with the interval $\mathfrak{Q}$-topology is a Hausdorff and weakly regular $\mathfrak{Q}$-enriched topological space.

This talk is based on [1] and is joint work with Javier Gutiérrez García (University of the Basque Country UPV/EHU) and Ulrich Höhle (Bergische Universität).

References

[1] I. Arrieta, J. Gutiérrez García, and U. Höhle, *Enriched lower separation axioms and the principle of enriched continuous extension*, Fuzzy Sets and Systems **468**, art. no. 108633, 2023.

[2] J. Gutiérrez García, U. Höhle, and T. Kubiak, *Basic concepts of quantale-enriched topologies*, Applied Categorical Structures **29**, 983–1003, 2021.

*This is joint work with Javier Gutiérrez García (University of the Basque Country UPV/EHU) and Ulrich Höhle (Bergische Universität).