Reconstructing the topology of algebraic structures: how and why

Michael Pinsker *

Technische Universität Wien, Austria
marula@gmx.at

Many mathematical objects are naturally equipped with both an algebraic and a topological structure. For example, the automorphism group of any first-order structure is, of course, a group, and in fact a topological group when equipped with the topology of pointwise convergence.

While in some cases, e.g. the additive group of the reals, the algebraic structure of the object alone carries strictly less information than together with the topological structure, in other cases its algebraic structure is so rich that it actually determines the topology (under some requirements on the topology): by a result of Kechris and Solecki, the pointwise convergence topology is the only compatible separable topology on the full symmetric group on a countable set. Which topologies are compatible with a given algebraic object has intrigued mathematicians for decades: for example, Ulam asked whether there exists a compatible locally compact Polish topology on the full symmetric group on a countable set (by the above, the answer is negative).

The reconstruction of the topologies of automorphism groups, endomorphism monoids, and polymorphism clones of first-order structures is primarily motivated by model-theoretic questions, but also has applications in theoretical computer science. In the case of automorphism groups, the question of the relationship between the algebraic and the topological structure has been pursued actively over the past 40 years. It turns out that many of the most popular automorphism groups, including that of the order of the rationals and of the random graph, have unique Polish topologies. Endomorphism monoids are algebraically not as rich, and often allow many different compatible topologies. We show, however, that there is a unique compatible Polish topology on the endomorphism monoids of the random graph, the weak linear order of the rational numbers, the random poset, and many more.

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*This is from a joint work with L. Elliott, J. Jonušas, J. D. Mitchell, Y. Péresse, and works with C. Schindler.